Pareto-improving Congestion Pricing on Multimodal Transportation Networks

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Outline

- Background
- Pareto-Improving Pricing Scheme
- Pareto-Improving Pricing Problem on Multi-modal Network
- Mathematical Model
- Numerical Examples
- Conclusion
Background

- **Congestion Pricing**
  - Alleviate traffic congestion by charging tolls.
  - **Current Practice:**
    - London, UK
    - Singapore
    - Stockholm, Sweden
  - Can successfully reduce congestion, but is still facing strong objection from the general public.
There are 3.6 travelers for OD pair (1, 3)
Example (Contd.)

User Equilibrium

System Optimal Under Marginal Cost Toll

Pareto-Improving

Total travel time: 255.82
1-2-3: 71.06
1-2-4-3: 71.06

Total travel time: 227.11
1-2-3: 101.70
1-2-4-3: 101.70
1-4-3: 101.70

Total travel time: 241.17
1-2-3: 69.46
1-2-4-3: 69.46
1-4-3: 69.46
Pareto-Improving Approach

- By Italian economist Vilfredo Pareto
- make at least one individual better off without making any other individual worse off
Single Modal Pareto-Improving Scheme

- Studied by Lawphongpanich and Yin (2008)
- Pareto-improving condition may be relatively prevalent
- Exact Pareto-improvement may not lead to significant travel time reduction.
Develop a Pareto-improving model for multimodal networks.

Allow cross-subsidy of different travel modes to encourage travelers switch to higher occupancy travel modes in order to further increase the system efficiency.
Multi-Modal Pareto Improvement (Contd.)

- Three travel modes:
  - Single Occupancy Vehicle
  - High Occupancy Vehicle
  - Transit (transit shares highway lanes with auto modes)

- Three types of transportation facilities:
  - Regular (Toll) Lanes
  - High Occupancy Toll (HOT) Lanes
  - Transit services (fixed frequency and capacity)
Multi-Modal Pareto-Improvement (Contd.)

- **Assumptions**
  - One class of homogenous users
  - Total O-D demand is fixed and known.
  - Users’ decision on travel-mode choosing follows logit model based on travel cost.
  - Traffic flow distribution follows user equilibrium condition within a chosen mode.
Multi-Modal Pareto-Improvement (Contd.)

- Objectives
  - The user utility will not decrease for any traveler.
    - Remain in the same travel mode and the travel cost will not increase.
    - Switch to another more preferred travel mode.
  - The total collected toll will be enough to cover all the transit subsidy.
  - The total social benefits will increase.
Modeling Transit User Behavior

- Waiting time
- Transfers

- Strategies
  - Boarding the first arrived vehicle within a selected subset in order to achieve shortest expected travel time.
Network Structure

- Original Network:

- Transit

  - Line a:
  - Line b:
  - Line c:
Network Structure (Contd.)

- **Modified Network:**

```
\begin{itemize}
\item Regular link: \text{(travel time, inf, 0)}
\item HOT link: \text{(travel time, inf, 0)}
\item Transit link: \text{(travel time, 0, transit capacity)}
\item Embarking link: \text{(boarding time, waiting time, inf)}
\item Alighting link: \text{(alighting time, 0, inf)}
\end{itemize}
```
Multi-Modal Pareto-Improving Model

\[
\max \sum_w \frac{1}{\theta} \cdot \ln \left( \sum_m \exp \left( -\theta (\rho_{j}^{w,m} - \rho_{i}^{w,m}) - \beta^m \right) \right) \cdot D^w + \sum_m \sum_l \tau^m_l \sum_w x^{w,m}_l \\
\text{s.t.} \\
\text{Tolled User Equilibrium Condition (1–10),} \\
-E^{w,m} \rho^{w,m} \leq t^{w,m}_{UE}, \quad \forall w \in W, m \in M, \\
\sum_w \sum_m \sum_l \tau^m_l x^{w,m}_l \geq 0, \quad \forall w \in W, m \in M, l \in L, \\
\tau^H_l = \tau^S_l, \quad \forall l \in L^S, \\
\tau^H_l = 0, \quad \forall l \in L^H, \\
\tau^H_l, \tau^S_l \geq 0, \quad \forall l \in L, \\
(x, d, \omega) \in \Phi.
\]
Multi-Modal Pareto-Improving Model (Contd.)

- Tolled User Equilibrium Condition

\[
(t_i(x) + \tau^m_i - (\rho^w_{j,m} - \rho^w_{i,m}))x^w_{i,m} = 0, \quad \forall l \in L, w \in W, m \in \{S \cup H\}, l \in L^+_i, l \in L^-_j, \quad (1)
\]

\[
(t_i(x) + \tau^T_i + \mu^w_i + \gamma_i - (\rho^w_{j,T} - \rho^w_{i,T}))x^w_{i,T} = 0, \quad \forall l \in L, w \in W, l \in L^+_i, l \in L^-_j, \quad (2)
\]

\[
\frac{1}{\theta} (\ln d^w_{m} + \beta^m) + \bar{\lambda}^w - E^w_{m} \rho^w_{m} = 0, \quad \forall w \in W, m \in M, \quad (3)
\]

\[
\sum_{\forall l \in L^T_i} f_i \mu^w_i = 1, \quad \forall i \in I, w \in W, \quad (4)
\]

\[
\gamma_i \left( \sum_{\forall w \in W} x^w_{i,T} - c^T_i \right) = 0, \quad \forall l \in L^T, \quad (5)
\]

\[
\mu^w_i \left( x^w_{i,T} - f_i \omega^w_i \right) = 0, \quad \forall l \in L^T, w \in W, \quad (6)
\]

\[
t_i(x) + \tau^m_i - (\rho^w_{j,m} - \rho^w_{i,m}) \geq 0, \quad \forall l \in L, w \in W, m \in \{S \cup H\}, l \in L^+_i, l \in L^-_j, \quad (7)
\]

\[
t_i(x) + \tau^T_i + \mu^w_i + \gamma_i - (\rho^w_{j,T} - \rho^w_{i,T}) \geq 0, \quad \forall l \in L, w \in W, l \in L^+_i, l \in L^-_j, \quad (8)
\]

\[
\gamma_i \geq 0, \quad \forall l \in L^T, \quad (9)
\]

\[
\mu^w_i \geq 0, \quad \forall l \in L^T, w \in W, \quad (10)
\]
Multi-Modal Pareto-Improving Model (Contd.)

\[
\max \sum_{w} \frac{1}{\theta} \cdot \ln \left( \sum_{m} \exp \left( -\theta (\rho_{j}^{w,m} - \rho_{i}^{w,m}) - \beta^{m} \right) \right) \cdot D^{w} + \sum_{m} \sum_{l} \tau_{l}^{m} \sum_{w} x_{l}^{w,m}
\]

s.t.
Tolled User Equilibrium Condition (1–10),
\[-E^{w,m} \rho^{w,m} \leq t_{UE}^{w,m}, \quad \forall w \in W, m \in M,\]
\[\sum_{w} \sum_{m} \sum_{l} \tau_{l}^{m} x_{l}^{w,m} \geq 0, \quad \forall w \in W, m \in M, l \in L,\]
\[\tau_{l}^{H} = \tau_{l}^{S}, \quad \forall l \in L^{S},\]
\[\tau_{l}^{H} = 0, \quad \forall l \in L^{H},\]
\[\tau_{l}^{H}, \tau_{l}^{S} \geq 0, \quad \forall l \in L,\]
\[(x, d, \omega) \in \Phi.\]
This problem is a mathematical programming with complementarity constraints (MPCC).

Solved by adapting manifold suboptimization algorithm proposed by Lawphongpanich and Yin (2008)
Numerical Example 1

O-D demand:

1-3: 30 1-4: 30
2-3: 30 2-4: 40
## Numerical Example I (Contd.)

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Numerical Examples II

- 90 links with 14 HOT links.
- 1 transit line on each link.
- 528 OD pairs.
Numerical Examples II (Contd.)

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<td>Max travel cost decrease (%)</td>
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<td>27.51 49.08</td>
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<td>0.68 0.68 39.78</td>
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Pareto improvement can be achieved in multimodal networks by adjusting the transit fares and charging tolls on highway links to redistribute traffic flows among travel modes and among the network.

The multimodal Pareto-improving tolls can be obtained by solving an MPCC problem using a manifold suboptimization technique.
Thank you!
Mathematical Model

- **Feasible Region**
  - Flow balance constraint:
    \[ Ax^{w,m} = E^{w,m} d^{w,m}, \quad \forall w \in W, m \in M, \]
  - Total OD demand:
    \[ \sum_{m \in M} d^{w,m} = D^w, \quad \forall w \in W, \]
  - Transit capacity:
    \[ \sum_{w \in W} x^{w,T}_l \leq c^T_l, \quad \forall l \in L^T, \]
Mathematical Model (Contd.)

- Feasible Region
  - Common-line:

\[ x_{i,w,T}^T \leq f_i \omega_i^w, \quad \forall i \in N, l \in L_i^+, w \in W, \]
Mathematical Model (Contd.)

- **Link Travel Time**

\[ t_l = \text{fft}_l \cdot \left(1 + 0.15\left(\frac{v_l}{\text{cap}_l}\right)^4\right), \quad \forall l \in L^H \cup L^S. \]

- Transit link travel times are the same with the auto link travel times that they share.
Mathematical Model (Contd.)

- **Total Link Vehicle Flow**

\[
v_l = \sum_w x_{l,w,S} + \sum_w \frac{x_{l,w,H}}{O^H} + \sum_{l' \in L_l} f_{l',r},
\]
User Equilibrium (UE) Condition within the Same Travel Mode

\[
C^{w,m}_p - t^{w,m} = \begin{cases} 
  0 & \text{if } b^{w,m}_p > 0 \\
  \geq 0 & \text{if } b^{w,m}_p = 0
\end{cases}, \quad \forall p \in P^{w,m}, w \in W, m \in M.
\]

- \(C^{w,m}_p\): path travel time on path \(p\) for OD pair \(w\) by mode \(m\),
- \(t^{w,m}\): the smallest travel time for OD pair \(w\) by mode \(m\),
- \(b^{w,m}_p\): flow on path \(p\) for OD pair \(w\) by mode \(m\).
Modal Split

The users’ mode choice behavior follow logit model.

\[ d^{w,m} = \frac{\exp(-\theta \cdot t^{w,m} - \beta^m)}{\sum_{m \in M} \exp(-\theta \cdot t^{w,m} - \beta^m)} \quad \forall w \in W \]

\( \theta, \beta^m \): parameters to be calibrated.
Multi-Modal User Equilibrium

Theorem. The solution of the following variational inequality (VI) problem (or MUE-VI) satisfies the multi-modal user equilibrium conditions.

\[
\sum_{w} \sum_{m} \sum_{l} t_l(x^*) (x_{l}^{w,m} - x_{l}^{w,m*}) \\
+ \frac{1}{\theta} \sum_{w} \sum_{m} (\ln d_{w,m*} + \beta^m) (d_{w,m} - d_{w,m*}) \\
+ \sum_{w} \sum_{i} (\omega_i^{w} - \omega_i^{w*}) \geq 0, \quad \forall (x, d, \omega) \in \Phi,
\]

\(\Phi\): the previously defined feasible region.
Multi-Modal System Optimum

- **Objective:** Maximize the total social benefits.

\[ \sum_{w} \frac{1}{\theta} \cdot \ln \left( \sum_{m \in M} \exp \left( -\theta \cdot t^{w,m} - \beta^{m} \right) \right) \cdot D^{w} \]

- **Multi-modal system optimal (MSO)**

\[
\begin{align*}
\min & \sum_{w} \sum_{m} \sum_{l} t_{l}(x)x^{w,m}_{l} + \frac{1}{\theta} \sum_{w} \sum_{m} d^{w,m} \ln d^{w,m} \\
& + \frac{1}{\theta} \sum_{w} \sum_{m} d^{w,m} \beta^{m} + \sum_{w} \sum_{i} \omega_{i}^{w} \\
\text{s.t.} & \ (x, d, \omega) \in \Phi.
\end{align*}
\]