A Competitive Facility Location Game with Traffic Congestion Costs

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Outline

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Motivation and Introduction

- Our aim in this study is to characterize the tradeoffs and costs associated with transportation congestion costs in supply chains
  - Analyze the effects of traffic congestion costs on supply chain operations
  - Provide methods to efficiently plan supply chains in presence of congestion costs

- Quantity and facility location decisions of a firm
  - In a competitive environment with multiple firms and multiple markets
  - Each firm may open multiple facilities
  - Firms are subject to traffic congestion costs due to their distribution volume decision
Literature Review

- Traffic Congestion Literature
  - Studied considering individual drivers, methods to decrease traffic congestion, traffic network equilibrium, congestion pricing, toll prices, signaling...

- Traffic Congestion and Supply Chains
  - Lot sizing and traffic congestion (Kamran et al. 1997), Effects of traffic congestion on supply chains and logistics (Figliozzi 2009, Jayaram et al. 2005, McKinnon 1999), Surveys with limited data and based on empirical data...

- Competitive Location Games
  - A set of profit maximizing firms competing to serve markets, Cournot competition (price is determined by the amount of supply), Different number of firms (2 firms, multiple firms), Different costs in the profit functions (homogeneous and heterogeneous)
Literature Review

- **Rhim et al., 2003**
  - Multiple firms and multiple markets, Cournot competition, homogeneous cost structure with linear transportation costs, each firm may locate a single facility.
  - Two-stage solution approach: first solve equilibrium quantity decisions for given locations of facilities, then heuristic search method for equilibrium location decisions (genetic algorithm).

- **Saiz and Hendrix, 2008**
  - The same settings but with heterogeneous cost structure.
  - The same solution approach; with a different heuristic search method for equilibrium location decisions (multi-start neighborhood search).
Problem Formulation

- A competitive facility location game with traffic congestion costs
  - Multiple firms and multiple markets, Cournot competition, *each firm may locate multiple facilities*, homogeneous and heterogeneous cost structures
  - Linear transportation costs and *convex congestion costs* (*quadratic convex cost function of total quantity flow on a specific link*)

- Solution Approach
  - Similar two-stage solution approach:
    - Solve the equilibrium quantities for given location decision
    - Heuristic search methods to find the equilibrium location decision
Problem Formulation

The profit function of a firm:

\[
\Pi_r(Q|X) = \sum_{j \in J} p_j \left( \sum_{i \in I} \sum_{r \in R} q_{ijr} \right) \sum_{i \in I} q_{ijr} - \sum_{j \in J} \sum_{i \in I} c_{ijr}(q_{ijr}) - \sum_{j \in J} \sum_{i \in I} g_{ijr} \left( \sum_{r \in R} q_{ijr} \right) - f_r(x_r)
\]

- Total Revenues
- Transportation Costs
- Traffic Congestion Costs

- Decreasing linear of the total quantity supplied to market,
- Linear transportation costs,
- Increasing Convex congestion cost function of total quantity on link (generally assumed to be quadratic).

For given location decision \((f_r(x_r) \text{ is constant})\):

- Profit function is (strictly) concave in each \(q_{ijr}\)
- The first order equilibrium conditions are independent of other market parameters and variables

Each market can be analyzed separately
Quantity Decisions: Stage-2 Game

The equilibrium conditions for supply quantities at a market can be defined as a variational inequality problem VIP

\[
- \sum_{r \in R_x} \sum_{i \in I_x} \frac{\partial \Pi^j_r(Q_j | X)}{\partial q_{ijr}} \times (q_{ijr} - q^*_{ijr}) \geq 0, \forall Q_j \in R_+^{i \times k \times x}
\]

**Theorem:** \( Q^*_j \) is an equilibrium solution if it solves

\[
\langle F(Q^*_j), Q_j - Q^*_j \rangle \geq 0, \forall Q_j \in \mathcal{K}
\]

**Proof:** Follows from concavity and equilibrium conditions. (See Nagurney 1999, Gabay and Moulin 1980)
Quantity Decisions: Stage-2 Game

- The properties of the variational inequality problem
  - $F(Q)$ is a linear function for quadratic congestion costs case
  - $F(Q)$ is monotone
  - $F(Q)$ is Lipschitz continuous
  - The VIP admits at least one solution (a unique solution when strategy sets are bounded)

- Modified projection method of Korpelevich 1977

- A self-adaptive projection method of Han 2006
  - Start with a $Q$, then update it by using projections and a weight which is adapted in each iteration
  - Both methods convergence to a solution
Market-Supply Game

- Assuming homogenous cost structure
  - **Proposition 1:** Facilities of different firms at the same location will supply the same amount to any market at equilibrium
  - **Proposition 2:** If a firm's facility at a location supplies a market, then other firms will also supply to the market from their facilities at that location at equilibrium

- Determine the quantities supplied from each location

\[ \Pi_i(Q|X) = \sum_{j \in J} p_j \left( \sum_{i \in I} Q_{ij} \right) Q_{ij} - \sum_{j \in J} c_{ij} Q_{ij} - \sum_{j \in J} g_{ij}(Q_{ij}) \]

- There exists a unique equilibrium solution (Rosen's theorem 1965)
- Characterization of the equilibrium solution:
  - for a special case
  - for the general case
Market-Supply Game: 

*Market-dependent Quadratic Congestion Costs*

- Congestion costs in many cases depend on the destination, and convex functions can be approximated by quadratic convex functions.
- We can solve the quantity decisions by sorting locations considering transportation costs (follows from KKT conditions) and solving first order conditions iteratively.
- Effects of congestion cost factor on the equilibrium quantities:
  - Increase in congestion cost factor of a market:
    - May increase the number of active locations at the market.
    - Decreases the total quantity supplied to the market.
    - There may be increase in the supply quantity for some locations even if the number of active locations increases and the total quantity supplied to the market decreases.

<table>
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<th>$Q_{i2}^*$</th>
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</table>
We can solve the quantity decisions by sorting locations considering transportation costs (follows from KKT conditions) and solving first order conditions iteratively.

**Effects of congestion factor and transportation cost:**
- Increase in congestion factor for a given location,
  - May increase the number of active locations at the market,
  - Decreases the quantity supplied from that location,
  - Decreases the total quantity supplied to the market,
  - Increases the quantities supplied from other locations
- Increase in transportation costs:
  - The number of active locations may increase or decrease or remain the same
  - The total quantity may increase or decrease depending on the active locations
  - Hard to trace!
Stage-one Game: Location Decisions

- We can solve quantity decisions given the location decisions for each firm.
- Our aim is to find the equilibrium location decisions.
- Dominated location decisions:
  - For a given location matrix $X$, if there exists a facility which does not supply, then this location matrix is dominated by the location matrix that does not have that facility.
  - For a given location matrix $X$, if there exists a firm with negative total profit, then this location matrix is dominated by the location matrix that has no open facility for that firm (viability).
# Heuristic Method

- **Start with a random location decision**
  - Move to a viable location decision
    - Rule 1: Close a facility, which has negative profit, for a firm with negative total profit and resolve (an heuristic operation)

- **For a given viable location decision**
  - Check equilibrium conditions
    - Rule 2: Close a facility of a firm that has negative profit and if an improvement observed update the random location matrix, if no improvement observed
    - Rule 3: Continue with full neighborhood search
Conclusions and Future Studies

- Our study combines traffic congestion and two main supply chain decisions: supply quantities and facility locations.

- The problem we study is a combinatorial problem and in current literature it is solved for very special cases under strict assumptions.
  - We consider a very general case and formulate it.
  - We provide efficient solution methods.

- The problem formulation allows us to analyze the effects of traffic congestion costs, as well as transportation costs, on two crucial decisions in supply chains of competing firms:
  - Quantity decision and Location decisions.

- Numerical studies on the methods and heuristics remain as a future work.

- It is observed that firms are affected negatively by traffic congestion costs.
  - Coordination between supply chain agents and government agents to reduce the effects of traffic congestion on supply chain efficiency.

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Thank You for Listening
Any Questions?