Optimal Dynamic Pricing Strategies for High Occupancy/Toll (HOT) Lanes

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Outline

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Introduction

- **Congestion Pricing**
  - First proposed in 1920’s (e.g., Pigou, 1920 and Knight, 1924)

- **HOT Lanes**
  - One form of congestion pricing
  - Converted from HOV lanes
  - Allows lower-occupancy vehicles to pay to gain access, but free for high-occupancy vehicles
  - First implemented in U.S. in 1995 (SR91 in CA), and now five in operation
  - I-95 in Florida coming soon (http://www.95express.com)

- **Objectives of HOT Lanes**
  - To provide a superior uncongested traffic service on the HOT lanes while maximizing the throughput rate of the freeway
Introduction (Cont’d.)

• **Dynamic Pricing for HOT Lanes**
  - One way to achieve the objectives of HOT lanes efficiently
  - Need to know
    - Current traffic condition and incoming flow
    - Willingness-to-pay (demand of HOT lane)
    - Traffic evolution after motorists react to the toll
  - Current practice
    - I-15 in California
    - I-394 in Minnesota

![Graph showing toll rate vs. time of day]
Introduction (Cont’d.)

• Limitations of Existing Methods
  – Practice: heuristic
  – Theoretical studies: ideal situations assumed
    • e.g., Arnott et al., (1998); Chu (1995); Liu and McDonald (1999); Yang and Huang (1997) and Kuwahara (2001)

• Research Objective
  – To explore possibilities of making dynamic pricing more effective and more attractive to both transportation authorities and motorists
Self-Learning Approach

To tune the demand function!
Based on the traffic dynamics!

Motorists’ Willingness to Pay

Calibrate
Optimize

Traffic Data from Sensors
Toll Rate
Calibration of Willingness to Pay

• **Logit Model**

\[
\frac{\lambda_T(t) - \mu_T(t)}{\mu_R(t)} = \frac{1}{1 + \exp(\alpha_1 (c_T(t) - c_R(t)) + \alpha_2 \beta(t) + \gamma)}
\]

• **System/Observation Equation**

\[
\ln\left(\frac{\mu_R(t)}{\lambda_T(t) - \mu_T(t)} - 1\right) = \alpha_1 (c_T(t) - c_R(t)) + \alpha_2 \beta(t) + \gamma
\]
Calibration of Willingness to Pay

• Estimation Algorithm
  – Recursive Least Squares (Kalman Filtering)

\[
\begin{aligned}
\begin{bmatrix}
\hat{\alpha}_1(t+1) \\
\hat{\alpha}_2(t+1) \\
\hat{\gamma}(t+1)
\end{bmatrix} &=
\begin{bmatrix}
\hat{\alpha}_1(t) \\
\hat{\alpha}_2(t) \\
\hat{\gamma}(t)
\end{bmatrix} + G(t)
\begin{bmatrix}
\ln\left(\frac{\mu_R(t)}{\lambda_T(t) - \mu_T(t)} - 1\right) - (c_T(t) - c_R(t)) \beta(t) \ 1
\end{bmatrix}
\begin{bmatrix}
\hat{\alpha}_1(t) \\
\hat{\alpha}_2(t) \\
\hat{\gamma}(t)
\end{bmatrix}
\end{aligned}
\]

\[
G(t) = P(t)
\begin{bmatrix}
(c_T(t) - c_R(t)) \\
\beta(t) \\
1
\end{bmatrix}
\begin{bmatrix}
(c_T(t) - c_R(t)) \\
\beta(t) \\
1
\end{bmatrix}^{-1}
\]

\[
P(t) = \left[I - G(t)\begin{bmatrix}
(c_T(t) - c_R(t)) \\
\beta(t) \\
1
\end{bmatrix}\right]P(t-1)
\]
Optimal Toll Determination

- Traffic Dynamics
  - Multi-lane *hybrid* traffic flow model (Laval and Daganzo, 2006) to explicitly describe the effect of lane-changing behaviors

\[ (q_m, q_n, k_l) = \psi(\alpha_1, \alpha_2, \gamma, \beta, \mu_T(t), \mu_R(t)) \]
Optimal Toll Determination (Cont’d.)

• Toll Optimization
  – Rolling Horizon Framework

\[
\begin{align*}
\max_{\beta} \ & \left\{ \sum_{j=t+1}^{t+N} [q_m(j) + q_n(j)] + \theta \cdot \min \left[ \tilde{k} - \frac{1}{N} \sum_{j=t+1}^{t+N} k_i(j), 0 \right] \right\} \\
\text{s.t.} \ & \quad 0 \leq \beta \leq \beta_{\text{max}}
\end{align*}
\]
Simulation Study

- Parameter Calibration
- Optimal Toll Rates

[Graphs showing calibration of parameters α1, α2, and γ, along with actual values, over time intervals.]
Simulation Study (Cont’d.)

• HOT Throughput

Average Throughput: 1149.6/1800 vphpl

• HOT Density

Average Density: 30.31 vplpm
Model Extensions

• Alternative HOT Lane Slip Ramp Configuration
  – Lane-changing behavior is largely affected by the physical configuration of the HOT access design
  – Three typical designs of HOT lane slip ramp

- Average Throughput: 1750.8/1800 vphpl
- Average Density: 40.55 vplpm
Model Extensions (Cont’d.)

- **Proactive Self-Learning Approach**
  - Fluctuating tolls may cause safety issues in reality
  - Predict short-term future inflows to adjust toll rates in a smoother manner

![Graph showing toll rate vs. time interval](image)

- **Scheme A: Reactive**
  - Average Throughput: 1018.8/1800 vphpl
  - Average Density: 16.98 vplpm

- **Scheme B: Proactive**
  - Average Throughput: 1075.8/1800 vphpl
  - Average Density: 33.86 vplpm
Model Extensions (Cont’d.)

• Demand Prediction
  – Assume traffic arrival follows Poisson process whose average rate is unknown

  \[ P(N = n|\Lambda = \lambda, t) = \frac{e^{-\lambda t} \cdot (\lambda t)^n}{n!} \]

  \[ f_\Lambda(\lambda) = \frac{a e^{-a \lambda} \cdot (a \lambda)^{k-1}}{\Gamma(k)} \]

  – Updating the distribution by Bayesian inference

  \[ f_{\Lambda|N(t)=i}(\lambda) = \frac{f(\lambda) \cdot P(N(t) = i|\Lambda = \lambda)}{\int_0^\infty f(\lambda) \cdot P(N(t) = i|\Lambda = \lambda) d\lambda} = \frac{(a + t) e^{-(a+t)\lambda} \left((a + t)\lambda\right)^{k+i-1}}{\Gamma(k + i)} \]

  \[ P(N(t + \Delta t) - N(t) = n|N(t) = i) = \int_0^{\infty} \frac{e^{-\lambda \Delta t} \cdot (\lambda \cdot \Delta t)^n}{n!} \cdot \frac{(a + t) e^{-(a+t)\lambda} \left((a + t)\lambda\right)^{k+i-1}}{\Gamma(k + i)} d\lambda \]

  \[ = \binom{k + i + n - 1}{n} \cdot \left(\frac{a + t}{a + t + \Delta t}\right)^{k+i} \cdot \left(\frac{\Delta t}{a + t + \Delta t}\right)^n \]
Concluding Remarks

• Self-Learning Approach for HOT Lane Operations
  – Step 1: recursive calibration of motorists’ willingness to pay
  – Step 2: rolling-horizon toll optimization
    • Multi-lane hybrid traffic model for modeling traffic dynamics

• Extensions
  – More realistic cell representations of HOT lane slip ramp
  – Proactive approach (coordination in time)
  – Demand prediction

• Future Research
  – Coordination in space
    • Localized v.s. system-wide / equity Issue
  – Heterogeneous users
Thank You!

QUESTIONS?