Final Report

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CENTER FOR MULTIMODAL SOLUTIONS FOR CONGESTION MITIGATION (CMS)

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Title: Multimodal Solutions for Large Scale Evacuations

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DISCLAIMER

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Report Summary

Introduction:

Summary of Problem:

In this research, a multimodal transportation model was developed attending the needs of emergency situations, and the solutions provided by the model could be used to moderate congestion during such events. The model incorporated features such as lane reversals as they have a significant impact on the evacuation efficiency. We proposed analytical techniques to solve the model. In this project, we developed a multimodal evacuation model and we hope to apply the model for an event management at the football stadium at University of Florida. The model is used to establish optimal evacuation routes to the bimodal evacuation problem and obtain efficient loading schemes for the staged evacuation. As a future extension, we are working on acceleration of the convergence of the algorithm.

Research Objectives and Scope:

In this research, we intend to establish optimal evacuation routes using multimodal transportation. The proposed solution procedure will also provide a loading scheme for the evacuation. The evacuation time obtained from the proposed techniques will be shorter than the contemporary evacuation techniques. We design analytical techniques to develop a staged evacuation procedure. Staged evacuation has empirically shown to handle congestion efficiently. Also, their impact is better realized under an emergency situation. Lane reversals will be implemented as a preprocessing step to further improve the efficiency. An integer program has been formulated to solve the lane reversal problem and it will be used to identify the lanes. We will also provide proof for polynomial time solvability of the lane reversal in maximum dynamic-flow and quickest flow problems. Multimodal flow problems are known to be NP-hard, a class of problems with no known polynomial time algorithm to solve them, making the problem under consideration quite significant. The optimization techniques currently employed could be formulated as a network flow problem, wherein the nodes of the network are the places, which could be a city or a house or a building room that are linked by the arcs, which includes roads, corridors or streets. Static demands and travel times will be considered as a simplistic assumption to make the model computationally efficient. The model developed will be implemented to manage an event in the football stadium of University of Florida at Gainesville. We have procured the necessary data and it requires some processing to fit the requirement of inputs of the model.

Progress Schedule

Table 1: Progress Schedule

Month	Planned gross Expenditure	Actual Gross Expenditure	Planned Progress (%)	Estimated Actual Progress(%)
1	10	10	7	7
2	14	14	18	18
3	24	24	26	26
4	31	31	45	45
5	40	40	58	58
6	47	47	70	60
7	61	61	75	62
8	70	65	83	65
9	80	70	88	70
10	85	75	94	75
11	95	80	97	77
12	100	85	100	80
13	100	85	100	85
14	100	90	100	85
15	100	100	100	100

Project Deliverable

Table 2: Project Deliverable

Task	*Description*	*OriginalDueDate*	*DateDelivered*
1	A comprehensive review of evacuation problem	First quarter	First quarter
2	Lane reversal Strategies and complexity of contraflow problems	Last quarter	First quarter
3	Complete model and implementation	Second quarter	Second Quarter
4	Testing on computer generated instances	Third quarter	Third quarter
5	Theory for stabilization and acceleration of convergence of algorithms	Fourth quarter	Fourth quarter
6	Testing of benchmark instances of planar and grid graphs	Fourth quarter	Fifth quarter
7	Implementation of Stabilization of techniques proposed	Fourth quarter	Fifth quarter
8	Real time instance test preparation & preprocessing	Fourth quarter	Fifth quarter

Overall Accomplishments of the Project

1. We carried out a comprehensive survey on evacuation models available in the literature.
2. We conducted a complexity studies and developed algorithms for various lane reversal strategies for network flow problem.
3. We developed a branch-and-price model for the bimodal evacuation problem.
4. We implemented the branch-and-price model in parallel computer and tested it successfully for some benchmark instances comprising of planar and grid graphs.
5. We developed the mathematical theory that involves numerical speed-up of the branch and price procedure. Some of these of have been implemented and some of them are currently being implemented or developed.
6. We tested some additional benchmark instances for planar graphs for multicommodity flow problems
7. The implementation of box stabilization technique has been carried out

We now provide the research findings in the following sections. In section 5, we provide a thorough survey on evacuation models that are available in the literature. In section 6, we formally state the contraflow problem, study the complexity of the problem and provide algorithms for simple cases. In section 7, we develop and test a bimodal evacuation model with buses and cars as modes of transportation. Finally, we summarize and conclude with a note on future work with possible extensions, validation and real-life applicability.

Review of Evacuation Problems

Hurricanes, earthquakes, industrial accidents, nuclear accidents, terrorist attacks and other such emergency situations pose a great danger to the lives of the populace. Evacuation during these situations is one way to increase safety and avoid escalation of damages. Evacuation problems are being given increased attention over the last five years. The techniques that are currently employed could be broadly categorized into optimization or simulation methods. In either case, the evacuation problem is dealt over a network where arcs or edges are the roads linking two places or the nodes of the network. The typical factors usually taken into consideration by these models are origin-destination assignment, response time of the evacuees, modes of transportation, contra flows etc. Factors such as origin-destination assignment and arc capacities could be set as static or dynamic, and these decisions heavily influence the evacuation efficiency. Most of the recent surveys and reviews in the field of evacuation were made for specific instances Gwynne et al. (1999); Santos and Aguirre (2004). The surveys cater to a specific technique and discuss in detail on their impact on emergency evacuation. This chapter is more across the board. The objective is to present a comprehensive report on the techniques that are available in the literature and broadly classify them. An initial classification was made depending on the approaches used in the evacuation models, namely optimization-based or simulation-based approaches. Furthermore, a subclassification was made based on some vital features considered by the model. These are features that are expected to have a significant impact on evacuation efficiency. The solution methodologies, for every model under these classifications, were discussed in detail and were also assessed based on their computational performance, scalability, extensibility, realizability and the major components considered. Computational efficiency of the model is important in case of unforeseen events. The models needs to be executed quickly to generate alternative plans and prepare for the dynamic scenarios. The complexity of the evacuation problems makes the researchers resort to for heuristic procedures and simulation-based approaches. Thus the solutions tested and provided needs to be scalable, or in other words, realizable on larger instances. Most of the models are customized to specific situations but when needed to accommodate additional features and handle more parameters they need to be extensible. The realizability of the model is achieved when they are tested on real time networks and most of the theoretical guarantees are accomplished. The chapter finally concludes with few words on the shortcomings of the available models that may require attention and lays down vital features that a new designer has to seek in a model.

Optimization Techniques

A large number of optimization models, sometimes referred to as analytical models in the literature, have been developed for evacuation studies.While some of these models are discrete, the rest are extensions from these base models. However, these models are essentially a simple network flow problem trying obtain a minimum cost flow from source to destination. A detailed discussion of the flow problems is carried out in the following section.

Maximum Dynamic Flow

An elementary evacuation flow problems could be formulated as a linear integer program using a variant of the maximum dynamic flow problem Ford and Fulkerson (1962, 1958). The maximum dynamic flow problem is to determine the maximum amount flow from origin to destination within a specific time T. Ford and Fulkerson formulated this problem on a time-expanded static network, where each node and edge is replaced by T copies corresponding to each time instance Ford and Fulkerson (1962). Given a digraph $G(V, E)$ and time interval T, let $c(u, v)$ and $t(u, v)$ be the capacity and traversal time of arc $(uv) \in E$. Let $x(u, v, \tau)$ be the amount of flow leaving node u along arc (u, v) at time τ. Let the origin node be s and destination node be t. Assuming holding over of flow over any node is allowed, we have the following integer linear programming problem that solves the maximum dynamic flow problem:

$$\text{Maximize} \quad \sum_{\tau=0}^{T}\sum_{(su)\in E} x(s,u,\tau) - \sum_{\tau=0}^{T}\sum_{(us)\in E} x(u,s,\tau - t(u,s)) \tag{1}$$

s.t.

$$\sum_{(uv)\in E} x(u,v,\tau) - \sum_{(vu)\in E} x(v,u,\tau - t(v,u)) = 0$$
$$, \ \forall\ u \neq s,t, u \in V, \tau = 0,1,\ldots,T \tag{2}$$

$$[\sum_{\tau=0}^{T}\sum_{(su)\in E} x(s,u,\tau) - \sum_{\tau=0}^{T}\sum_{(us)\in E} x(u,s,\tau - t(u,s))]$$
$$+[\sum_{\tau=0}^{T}\sum_{(tu)\in E} x(t,u,\tau) - \sum_{\tau=0}^{T}\sum_{(ut)\in E} x(u,t,\tau - t(u,t))] = 0 \tag{3}$$

$$0 \leq x(u,v,\tau) \leq c(u,v), \forall (uv) \in E, \tau = 1\ldots T \tag{4}$$

The objective function that has to be maximized gives the net amount of flow leaving the origin node s by time period T. The constraint set (2) ensures conservation of flow at every node, where the amount of flow that enters a node is exactly equal to the amount of flow that leaves the node at any time period. The constraint set (3) ensures that at the end of T time periods the amount of flow that leaves the origin s is equal to the amount of flow that enters the destination t. The above formulation solves a maximum flow problem over a time expanded graph. The problem becomes extremely difficult to solve for larger graphs with a bigger time frame. However, in Ford and Fulkerson (1962), the authors suggested a strictly polynomial algorithm for the maximum dynamic flow. They solved the minimum cost flow problem on the original graph, without time expansion, and decomposed the flow into a set of paths. Then they obtained the maximum dynamic flow by temporally repeating the flow along the paths.

Most of the evacuation problems are ramifications of the maximum dynamic flow problem. The quickest flow problem, sometimes referred to as evacuation problem, is to determine the minimum time required to send a given amount of flow from the origin to the destination. This problem is a simple variation of the maximum dynamic flow problem and could be solved through a binary search. In Burkard et al. (1993a), the authors proved this reduction from the maximum dynamic flow problem to the quickest flow problem and provided a strongly polynomial time algorithm through a parametric search on the quickest time by repeatedly solving the maximum dynamic flow problem. Many other work have been done in this line generalizing this concept. In Fleischer and Skutella (2002), a work on multicommodity shipment of flows was provided. A multicommodity flow problem on a static graph without a time bound is: Given a graph with a arc travel time and capacity on each arc and a set of commodities $K = 1,\ldots,k$ with each commodity having specific origin s_i and destination t_i, it is required to send a specific amount of flow from s_i to t_i of the corresponding commodity in the minimum amount of time. The problem is formulated in the following manner.

$$\text{Minimize} \quad \sum_{i=1}^{k} \sum_{(uv)\in E} c(u,v,i)x(u,v,i) \tag{5}$$

s.t.

$$\sum_{(uv)\in E} x(u,v,i) - \sum_{(vw)\in E} x(v,w,i) = 0, \forall v \in V, i = 1,\ldots,k \tag{6}$$

$$\sum_{(ut_i)\in E} x(u,t_i,i) - \sum_{(t_iv)\in E} x(t_i,v,i) \geq d_i, \forall i = 1,\ldots,k \tag{7}$$

$$0 \leq x(u,v,i) \forall uv \in E, i = 1,\ldots,k \tag{8}$$

$$\sum_{i=1}^{k} x(u,v,i), \leq c(u,v) \forall (uv) \in E, i = 1,\ldots,k \tag{9}$$

$x(u,v,i)$ is the amount of flow on arc (uv) of commodity i and $c(u,v,i)$ is the cost of unit flow on arc (uv) of commodity i. The constraint set (6) implies the flow conservation at a node for a particular commodity and the constraint set (7) ensures that the sinks node t_i receives d_i amount of flow. Finally, (8) and (9) restraints the flow capacity on each arc. The above is a multicommodity flow problem for a static graph. In order to solve the quickest multicommodity problem, we have to extend the formulation to a time-expanded graph similar to the previous problem formulation. However, this results in numerous constraints and variables if the time horizon, T, considered in large. Each node and arc of the graph is replaced by T copies, each corresponding to the specific time instant. The multicommodity flow problem is known to be NP-hard even for a static graph Gary and Johnson (1979). To overcome the time expansion difficulty a scaling algorithm could be employed, where in each node and arc is replaced by T/δ copies instead T, thus reducing the problem size considerably and striking a balance between the precision of the result and the running time of the algorithm Fleischer and Skutella (2002). In Hoppe and Tardos (1995), the authors provided solutions for three variations of maximum dynamic flow problem. They provided a polynomial time approximation for the earliest arrival flow problem, which was studied by Wilkinson (1971) and Minieka (1973). The earliest arrival flow problem requires the flow to be maximized at each time step of the given horizon, unlike the maximum dynamic flow problem. The proposed algorithm is based on successive shortest path algorithm, where the flow is augmented along the shortest path, quickest path in our case, and the chain decomposition is given by augmentations performed in a sequence of static graphs. However, successive shortest path is a pseudo-polynomial algorithm. This difficulty is usually handled by scaling. Unlike traditional scaling, the proposed algorithm performs an upward capacity scaling. A dynamic flow quickest path with a small capacity could be repeated temporally to obtain a maximum flow. We refer to Hoppe and Tardos (1995) for a detailed account of the algorithm and proof of approximation. The second problem studied was lexicographic maximum dynamic flow. Given a set of sources and their priority of evacuation the lexicographic maximum dynamic flow maximizes the flows leaving the sources in the specified order. Finally, they studied the quickest flow problem with fixed number of sources and destinations with equal priorities having specific supply demands. In Klinz and Woeginger (2004), the authors pointed out that the problem of determining quickest flow from any subset of nodes to any other subset of nodes is equivalent to a single source shortest path problem. Thus, for a fixed number of sources and destinations, this problem is polynomially solvable, if we consider all possible subsets of the sources and destinations. In the same fashion, for a fixed number of source and destination, the lexicographic maximum dynamic flow could be solved for all possible ordering of the sources to solve the above problem. A detailed and more efficient algorithm was given in Hoppe and Tardos (1994) for this quickest transshipment problem with more than one origin and destination. These models based on maximum dynamic flow problem have their practical limitations, as they are oblivious to factors such as congestion control, dynamic origin and destination demand and multimodal transportation, which are quite conceivable in a real-time situation. The flow problems are rather simplified versions of real time model. The models are computationally efficient with polynomial time solutions for exact or approximate solutions. The problems thus could help in providing quick solutions to large problem sizes under a simplified setting. The realizability of the model is limited but their computational efficiency could be taken advantage of in preprocessing stages of analytical techniques and heuristic developments.

Dynamic Traffic Assignment

In Merchant and Nemhauser (1978), the dynamic traffic assignment (DTA) problem was introduced and formulated as a non-linear program with a non-convex mathematical program. The problem is to find an assignment of flows on the links optimally. Like before, let us assume the planning horizon to be T. And $G(V, E)$ be the digraph under consideration. Let $x(u, v, \tau)$ be the amount of flow on arc (u, v) at time τ. Let $F(u, \tau)$ be the external input in node u at time τ and $h_{uv,\tau}(x(u, v, \tau))$ be the cost function. Also, let $g_{uv}(x(u, v, \tau))$ be the exit function denoting the amount of flow that exits from arc u,v during period τ. Finally, $d(u, v, \tau)$ be the amount of flow entering arc (uv) at node u. The DTA problem is formulated as follows:

$$
\begin{aligned}
\text{Minimize} \quad & \sum_{\tau=0}^{T} \sum_{(uv)\in E} h_{uv,\tau}(x(u, v, \tau)) && (10)\\
\text{s.t.} \quad & \\
& x(u, v, \tau + 1) - x(u, v, \tau) + g_{uv}(x(u, v, \tau)) \\
& \quad -d(u, v, \tau) = 0, \forall (uv) \in E, \tau = 1, 2, \ldots, T && (11)\\
& \sum_{\forall uv \in E} d(u, v, \tau) - F(u, \tau) \\
& \quad - \sum_{\forall pu \in E} g_{pu}(x(p, u, \tau)) = 0, \forall u \in V \backslash t, \tau = 1, 2, \ldots, T && (12)\\
& \quad 0 \le x(u, v, \tau), \forall (uv) \in E, \tau = 0 \ldots T && (13)\\
& \quad 0 \le d(u, v, \tau) \forall (uv) \in E, \tau = 0 \ldots T && (14)
\end{aligned}
$$

This work by Merchant and Nemhauser was the opening to dynamic traffic assignment. They formulated discrete piecewise linearized version of the traffic assignment problem. The problem assumes that the demands are available and it has a single destination. In most of the emergency situations these assumptions are not preserved. For instance, of a multicommodity flow where it is required to maintain the origin-destination pair, a single-destination model may not be suitable. Also, dynamic demand in place of a static demand estimate is more efficient and more precise in a real-time situation. In Carey (1986a), the model was validated by proving that the constraints satisfy linear independence constraint qualification as the exit function is continuously differentiable. In Ho (1980), the authors linearized the exit and cost function and obtained the global optimum for the nonlinear, non-convex optimization problem by solving a series of T+1 optimization problems. A link flow non-linear mixed integer programming formulation and a convergent dynamic algorithm to solve the dynamic user equilibrium problem was provided in Carey (1986b). It explicitly seeks equilibrium in terms of path travel times unlike Merchant and Nemhauser's model. The model depends on static use equilibrium functions with additional constraints to ensure temporally continuous flow. The technique itself is not pertinent to evacuation studies, and hence we refer to the work in Peeta and Ziliaskopoulos (2001) who made a detailed study on dynamic traffic assignment in their survey.

We are more interested in its applicability in emergency situations; however, the little background provided is required.

The cell-based dynamic traffic assignment that segmented the highway links into equal sized cells, such that each cell could be traversed in an unit time was provided in Daganzo (1994). Each cell has a specific capacity and the congestion is explicitly handled by restraining the amount of flow from one cell to another. In Ziliaskopoulos (2000), Daganzo's formulation was relaxed by holding of flow at nodes in the flow conservation equations. In Chiu et al. (2007), the authors implemented a dynamic traffic modeling technique based on the cell transmission model Daganzo (1994) for an optimal no-notice mass evacuation. They also proposed a network transformation to a single destination network and then a cell network, for the above implementation. Their objective in no-notice evacuation also includes identification of destinations. The modeling is done through a graph transformation. The model employs aggregation of zones and hence could be scaled appropriately to handle large graphs. The model assumes prior knowledge of originating demands and zonal information, which in most cases are available. Also, the optimization formulation was provided for a cell-based transmission for a time expanded graph. The model optimization model as such might be time consuming while applied over real time graphs. The authors pointed out that without user-equilibrium constraints the model may not be of practical interest. The model might be useful during no-notice mass evacuation, assuming that there is an efficient technique to solve it, but the assumptions make it rigid to extend it to other emergency situations. Another work employing cell-transmission model is in Liu et al. (2006a). They discuss the staged evacuation procedure, where in a zonal classification of the nodes is done based on the severity of impacts they suffer and starting time for each evacuation zone is determined, taking the response time of evacuees into consideration. These models based on cell-based transmission involves in a formulation for a time-expanded static graph with each link replaced by a group of cells. This methodology may not yield quicker results for an evacuation problem considered over a larger space and time, and this may be a necessity in an emergency situation. The complexity of the models will further increase when they consider more complex features, such as user-equilibrium constraints, congestions management, dynamic demand, etc. This computational difficulty could be overcome by aggregating the nodes of the network under consideration and solving the problem in a smaller graph. The efficiency could also be improved by increasing the coarseness of the discretization of time depending on the accuracy of the results desired. The models suggested an evacuation procedure with single destination or sink, which cannot handle multicommodity flows to identify optimal destinations. In Golani and Waller (2004), the authors suggested a heuristic to the cell-based transmission model with multiple destinations to overcome this implementation difficulty. However, the accuracy of the heuristic and its convergence when the problem size grows is a question of consideration. In Chalmet et al. (1982), the concerns in a time-expanded evacuation problem are provided in detail. Time-expanded networks, while having computational limitations in large-scale evacuations, might benefit the small-scale networks such as the building evacuation as the network under consideration is quite small compared to large scale evacuation networks. A pseudopolynomial algorithm for solving the maximum dynamic flow and quickest flow problems with a time-varying travel time, node and arc capacity was provided in Miller-Hooks and Patterson (2004). We refer to Kuligowski and Peacock (2005) for more details on small-scale evacuations, where they made a comprehensive review on the building evacuation models.

The variational inequality approach is another way of formulating the the DTA problem Bliemer and Bovy (2003); Ran and Boyce (1996); Peeta and Ziliaskopoulos (2001); Ban et al. (2005). Variational Inequality provides a convenient formulation technique for network equilibrium problems arising in economics, finance and transportation. It was put forth by Hartman and Stampacchia (1966) to study partial differential equations problems. In Dafermos (1980), the authors studied the equilibrium problems in transportation networks applying variational inequality for a static travel demand. A finite dimensional variational inequality problem could be stated as: Given a subset $k \subset R^n$ find a vector $x^* \in k$ such that

$$\langle F(x^*), x - x^* \rangle \geq 0, \forall x \in k \tag{15}$$

where $F : k \rightarrow R^n$ is a continuous function. In case of the transportation problem, the F is the cost function and x is the flow on an arc and n being the number of arcs in the network. The equilibrium conditions could be viewed as Kuhn-Tucker conditions for a convex nonlinear problem. In Nagurney (1999), a detailed account on variational inequalities and their applications to several equilibrium problems is given. A much wider range of problems including multimodal transportation and elastic demands were discussed along with a discussion of algorithms to network equilibrium problems formulated with variational inequalities. In Friesz et al. (1993), a variational inequality formulation for a continuous time problem with dynamic route choices and departure time decisions, a dynamic version of the static Wardropian user equilibrium, was provided. It considers the path costs realized through travel time and penalty for early or late arrival time and the optimal flow pattern is simultaneous route departure equilibrium. As the model is dealt in a continuous time domain, it might lack a good algorithm to efficiently solve complex integrals on the path costs presented in the model. Another continuous time model for transportation networks was provided in Raciti and L.Scrimali (2004). They considered a single commodity problem with elastic demand and a time-dependent equilibrium and expressed the corresponding equilibrium problem as quasi-variational inequality. The subset $k \subset R^n$ in equation (15) is replaced by a point-to-set map $K : E \rightarrow R^n$. Brotcorne et al. (2002) presented another variational inequality based equilibrium problem for a multicommodity flow problem where the user's desired origin-destination pair and departure and arrival time window is provided. A detailed section of variational inequality models in the transportation framework was provided Ran and Boyce (1996).

Non-Deterministic Methods

Significant work has been done on evacuation problems from non-deterministic perspective. Smith et. al have done substantial work on stochastic evacuation network Smith (1993); Smith and Towsley (1981); Smith (1987). They provide non-inferior solutions to bi-objective routing problem in queueing networks. The two objectives modeled were the distance traveled and the clearance time. The problem also considered the multicommodity flow problem. They provided a path based bi-objective formulation with flow conservation and capacity constraints for all the routing alternatives. The routes are generated and fed to the above formulation by an algorithm that iteratively generates candidate paths to assess congestion. In Talebi and Smith (1985), gave an overview of deterministic, stochastic and hybrid methods for evacuation problems and compared a analytical queueing network with a simulation model for a hospital evacuation considering evacuation time, congestion and optimal routes. In Opasanon (2004), two solutions were provided, one with optimal evacuation route and another with set of strategies from which the evacuee can choose the best arc at each step. They dealt the evacuation problem on network with stochastic, time-varying travel time. In Waller and Ziliaskopoulos (2001), the authors considered a stochastic dynamic network in which the origin destination demand are random variables with known probability distributions. A linear programming formulation based on the system optimal dynamic traffic assignment propagating traffic by cell transmission was provided. The results are robust as they build a confidence level which requires the solutions to meet the expected demands. In Ozbay and Ozguven (2007), the authors incorporated capacity constraints in cell transmission model for system optimal dynamic traffic assignment. The capacity constraints are probabilistic in nature due to the impacts of the disaster. They also compared the model to one with a deterministic capacity. In Bakuli and Smith (1996), the resource allocation problem was studied in the emergency evacuation network. They employed queuing theory to model the varying capacities circulation spaces, such as finite-sized corridors, staircases etc. and study their effects on throughput. The evacuees in this model experience a service at the evacuation nodes and this service rate decays with the increasing amount of traffic.

Significant Features in Optimization Techniques

This section will discuss the factors that are widely regarded to influence the evacuation efficiency. While some of them are only germane to optimization techniques, the rest will be revisited when we discuss the simulation techniques.

Contra Flows

A contra flow or reversible flow is a strategy to improve the efficiency of evacuation by allowing traffic in the opposite direction of the roads during an emergency. Most of the recent evacuation plans proposed incorporates contra flow in them. The contraflows while being efficient in evacuation they come with a few pitfalls like increase in accidents and operation cost, which might be a consideration for their implementation in the evacuation plans Bretherton and Elhaj. (1996).

Consider the following lane reversal problem: Given a directed graph $G(V, E)$ with non-negative arc capacities and travel times, a subset of nodes S as sources and planning period or horizon T, determine the quickest time to evacuate all the occupants from source to destination by allowing lane reversals. A single source, single sink contraflow problem is polynomially solvable, but a multiple source and sink problem becomes strongly mathsfNP-hard Rebennack et al. (2008). Thus, even simple cases of the problem is hard to solve and researchers have to resort to heuristic. Simulation procedures could be performed for a given configurations of networks and hence may not be a suitable tool to identify optimal lane identification, but the analytical procedures are not computationally efficient even for simple models without complex features Wolshon et al. (2001).

In Kim and Shekhar (2005), the authors proposed two heuristics to solve the evacuation problem with contra flow more than one source and destination. The first heuristic was proposed for a time-expanded static graph $G_T(V_T, E_T)$. The heuristic runs on a time expanded graph, making it less attractive for large-scale evacuation with large number of nodes and with evacuation planned over a bigger time frame. The second heuristic proposed was a simulated annealing procedure. The algorithm begins with solution and perturbs it a little to obtain a better solution and this iterative procedure is carried out until a stopping condition is satisfied. This is a standard technique employed to escape local minimum and obtain values close to global minimum. A similar tabu-based heuristic algorithm to solve lane-reversible evacuation problem was proposed in Tuydes and Ziliaskopoulos (2006b). It is a local search procedure attempting to get the best solution in a given neighborhood. The system optimal dynamic traffic assignment model for the cell-based transmission, which was described in section 2.2, is considered for the implementation of contraflows and it permits partial capacity reversal, unlike the former approach. In another work Tuydes and Ziliaskopoulos (2006a), they formulated the dynamic traffic assignment model with lane reversible capabilities. This information could be used as an efficient initial solution for the tabu-based search. The models discussed are simple flow problems with lane reversal capability and is far from being practically realizable. The models could however serve as a good preprocessing technique to identify the lanes to be reversed and then a more complex model could be employed on a reconfigured network. The heuristics developed to solve them, though computationally efficient, does not have any theoretical guarantee in the quality of the solutions. Thus the issue of scalability is a concern, as the quality of the solution could be compromised with the problem size. Approximate solutions to the contraflow problems for such guarantees is still an open area of study.

Evacuee Behavior

Like contra flows, another factor that is incorporated in the evacuation models in the recent years is the the response time of the evacuees. The response time is the time taken for a decision by the evacuee to evacuate. This latency could be due to the inefficiency in the information dissemination, panic in evacuees or the emergency situation itself. Conventional models disregard this, assuming zero loading and unloading time. In Sattayhatewa and Ran (2004), the authors formulated a nuclear power plant evacuation through bilinear programming and incorporated the response time of the evacuees. They used the formulation in Ran and Boyce (1996) for dynamic traffic assignment. They proposed a departure model with the objective of quickest evacuation time with one model departure time from origin and another model arrival time at the destination. In Helbing et al. (2000), the authors simulated the escape panic situation in which the clogging, jamming at widening, panic initiation and impatience are captured. This would help in validating an evacuation model developed without considering human factors. However, the study was aimed to model the pedestrian behavior, injuries caused due to congestion, uncoordinated passing of bottlenecks and physical interactions among people. Thus careful scaling of parameters is required if we need to realize this in larger network with vehicular motions. The model would more serve as a tool for calibration than for establishing optimal routes of evacuation. The survey about human behavior in fire Bryan (2000) gives a gist of the influence of evacuee behavior on models. The study caters specifically to the situation caused by fire and certain concerns raised, such as smoke effects, fire alarms, cannot be extended to a generic situation. This would help in a similar fashion as the previous model in setting standards for an evacuation model and features to be identified in a model. A model was developed in Fu and Wilmot (2007) to predict the response behavior of hurricane Andrew in Louisiana and they were found to be similar to hurricane Floyd in South Carolina. The response curve could be used to model response behavior in evacuation models. Since the study was performed and tested for behavior during hurricane, response behaviors during situations such as event management, nuclear accidents, fire hazards, and no-notice evacuations needs to be analyzed in order to assure its generic applicability. Chen and Zhan. (2006) provided the measures for efficiency of an evacuation plan. Among the various factors they considered, the loading curve of the time taken for evacuees to decide to evacuate was considered as a major factor, as this is crucial in determining the travel time of the individual evacuees. In Murray and Mahmassani (2004), the authors suggested that the panic situation as result of a danger results in the evacuees to move at random before restoring order, especially when they want to find the family members before trying to reach safety. They proposed a two-stage formulation to establish household trip chain sequence. In the first phase, the optimization model determines a meeting location for the family, and the second phase comprises of the trip assignment to evacuate people to safety. The strategies are later supplied to a simulator to study the effects of reassignment. It provides a different perspective to behavior modeling but not the only feature to be captured.

In case of a large-scale evacuation system, where the travel time is much larger compared to the response time of the evacuees, the error in the case of excluding the response time would be very minimal. However incorporating it might increase the complexity of the optimization model and hence decrease the computational efficiency. The implementation in two different phases, like that of contraflows, in order to overcome this problem could also be difficult as the behavior modeling dynamically changes the traffic flow. This is should be a another factor taken into consideration while developing a model. Sometimes behaviorial response is related closely to the situation and may be different at different times.

Dynamic Origin-Destination Demands

An observation on the most difficult obstacle before deploying DTA is estimating the time dependent demand was made in Peeta and Ziliaskopoulos (2001). The demand between origin-and destination in an evacuation problem is typically subject to the environmental changes. For instance, an evacuation scenario, which just requires the evacuees to reach safety, might not route passengers to the initially designated destination, as this would compromise the evacuation time. This could be because of any unforeseen circumstances such as congestion, road blockage, lane reversals or unavailability of shelter. Most researchers have assumed to have origin destination data available a priori and they don't change over time. Later, the analysis was carried out on a static demand. In Yuan et al. (2006), the authors formulated an evacuation problem where the origin destination pairs are not fixed. Given a set of origin or destination, the objective is to ship all the commodities from the origin set to any subset of destination nodes. This is more intuitive in an evacuation process, where the safety of individuals is the priority. This could be achieved through a simple transformation in which a dummy super-sink node is added with zero cost and infinite capacity arcs from all other destinations. Given an directed graph $G(V, A)$ and set of origins I and destinations J with arc capacities $c(a), \forall a \in A$ and cost function $h(x(a))$. Let D_{ij} be the demand matrix between OD pair. Now we have the following static demand evacuation problem.

$$\text{Minimize} \quad \sum_{\forall a \in A} h(x(a)) \tag{16}$$

$$\text{s.t.}$$

$$x(a) = \sum_{\forall r \in R} \delta_{ar} p_r, \forall a \in A \tag{17}$$

$$\sum_{\forall r \in R_{ij}} p_r = D_{ij}, \forall i \in I, j \in J \tag{18}$$

$$0 \leq x(a) \leq c(a), \forall a \in A \tag{19}$$

R is the set of all paths between all origins and destinations, p_r is the flow along the $r^t h$ path and δ_{ar} is indicator variable with value 1 is arc a is in path r and 0 otherwise. Constraint set (18) will be replaced by

$$\sum_{\forall r \in R_i} p_r = D_i, \forall i \in I \tag{20}$$

for the single destination.

This is another formulation for a time-expanded static graph, having all the drawbacks, with regards to computational efficiency, mentioned earlier. The model when assuming independent evacuees as flow variables, assume that the different family members could reach different destinations. As a path-based model has been presented, the scalability of the model needs more examination as the analytical techniques using path or route-based modeling needs to consider exponentially many paths available. The sequential logit model was applied to the model the dynamic traffic demand for evacuation problems with the underlying assumption that the demand varies with the evacuee behavior Fu and Wilmot (2004) . The time horizon is discretized and the decision to evacuate at each time interval is modeled as a sequence of binary decisions. Later the probability that a household will evacuate at a specific time is calculated using random utility theory, which in turn is used to estimate time varying demand. The problem of updating origin-destination matrix based on the traffic link count has also been studied in Stathopoulos and Tsekeris (2004); Zhou et al. (2006). The formulation allows traffic flow according to the traffic conditions and they are brought under an entropy maximization framework. Another useful study in this line was in Yuan et al. (2007), where they compared two models of compliance and non-compliance to predetermine the route and destination assignments for evacuation using simulation software. It was proposed in Theodoulou and Wolshon (2004), a genetic algorithm to estimate the dynamic demand between the origin destination pair. The method is based on the fact that dynamic demand OD matrix depends on spatial and temporal variations of congestion, or in other words, the variation in demand is based on the traffic count on a link. The congestion of link and hence a specific path from an origin to destination would cause the optimal solution to seek an alternate path causing an alteration in demand. The genetic algorithms tries to find the optimal OD demand matrix from the seed OD matrix provided as input.

Multimodal Transportation

Multimodal transportation is realistic in an emergency situation and it also considerably affects the evacuation efficiency. It is therefore essential to incorporate mode choices in the models in order to develop precise models. A formulation based on cell transmission provided in Chang and Ziliaskopoulos (2004) permits intermodal transportation with cars and buses for a single destination network. The formulation was a system optimal integer linear programming. Thus the corresponding flow problem is a bimodal transportation for a single commodity. The model assumes no pedestrian movements, but transit cells are considered where evacuees can move to another vehicle. The model is a very basic model and has several assumptions that makes it practically infeasible. For instance, the model assumes that there is an unlimited supply of buses and cars. Also, the model contains enormous amounts of integer variables and constraints. Addition of multicommodity flow constraints would make the model even more complex. In Blum and Eskandarian (2004), the authors studied the influence of multimodal transportation on the evacuation efficiency in building complexes. They extended the cellular automata model, where space is divided into cells and time is discretized such that vehicles can move from one cell to another in one time step, to capture the interaction between pedestrians and vehicle behavior. To overcome the computational complexity posed by the formulations in the optimization models, investigators explored heuristic algorithms to solve the evacuation problem. In Liu et al. (2007), the authors proposed a heuristic algorithm, HASTE, which provided a close approximation to the optimal solution that could be achieved through a linear program for the cell based dynamic traffic assignment problem. This overcomes the difficulty of the computational time spent on the overwhelming size of the problem at a price of approximating the optimal solution. The accommodation of multimodal transportation usually increases the precision of the model as it is more realistic and also the evacuation efficiency. However, it increases the complexity of the model as a whole and decreases the computational efficiency.

Miscellaneous Factors

The key logistical issues during the aftermath of a disaster have been carefully analyzed in Holguin-Veras et al. (2007). The response and recovery operations such as supply of food and medicine on reduced capacity network are discussed. The research was based on public accounts and interviews made during the field visits to Hurricane Katrina-impacted areas. Another interesting study in the context of evacuation is to examine the vulnerability of the network and identify the insecure links in the network. It is important because it accounts for the connectivity between the origin and destination during evacuation. This may allow us to estimate the arc or link capacities of the network for evacuation. In Tuite and Mahmassani (2004), the vulnerability of a network using a vulnerability index was studied and it was aggregated over all origin destination pairs. A topological index to determine the depressiveness/concentration, which helps in identifying isolated districts, was provided Sakakibara et al. (2004) and estimates the robustness of the road network in an emergency situation. The travel choice is tied to the vulnerability of the network. The change in the demands and flows owning to disruptions in the network was also studied in Chen et al. (2007). They formulated a travel demand model to derive a measure to assess the vulnerability of a transportation network. The measures are capable of valuating the changes in both demand and supply. In Chen and Zhan. (2006), the authors used an agent-based simulation technique, PARAMICS, to compare the staged and simultaneous evacuation. They concluded that for a general network topology there is no specific strategy that results in better evacuation efficiency. However, a high-density population could be evacuated quicker over a grid-structured network by staged evacuation.

Simulation Techniques

The large size and time over which the optimization models have to be implemented make them less suitable for immediate realizability. Simulation-based approaches for evacuation strategies are widely adopted to overcome this practical difficulty. Also, simulation-based models could be a tool to study the current plans without actually executing it. The evacuation plans generated by the analytical models discussed above could be simulated to identify inconsistencies in the model, thus serving as a good instrument for validation. The simulation models permit the designers and researchers to visualize the evacuation and hence it is more incisive compared to analytical models.

One level of breakdown of the simulation-based technique will be as macro, micro or meso traffic simulations. A microscopic traffic model captures the lineaments of individual vehicles, whereas a macroscopic traffic model provides a collective vehicle dynamics. Macroscopic simulations are similar to fluid flow study, where the estimates are based on a group of vehicles as a whole. Macroscopic simulations are not computationally expensive, but fails to adapt to random and rapid changes in the environment. Micro simulations is a diametric simulation technique, where the characteristics of individual vehicles are captured and thus able to predict and adapt to the changes in the model more efficiently than macroscopic simulations. The disadvantage of this technique is the computational expense encountered as the system size grows and more vehicles are added to the system. However, the micro-simulators are getting popular in the recent years with the increase in the computational capacity. A detailed note about macro and micro simulations and their relative advantages and disadvantages is discussed in Pidd et al. (1993).

Microscopic Simulation Techniques

Agent-based simulations are also called as micro-simulators, as the agents or drivers involved in the system could be studied individually. In most traffic simulation studies there is a need for the simulation to be performed with more precision. For instance, vehicular interactions during congestion lane change or traffic signals at intersections could be dealt only in a microscopic model. In Cova and Johnson (2002), the authors performed an agent-based neighborhood evacuation study. The model has a nice property to study the disaggregate outputs such as the vehicle safety and travel times within zones rather than average travel time of entire system. However, some of the assumptions are rigid such as departure times were assumed to be available and the destinations were preassigned. These assumptions may not be preserved in real-time situation. The topology of the tested network was relatively simple compared to real-time networks. It will be interesting to compare the computational efficiency when tested real time with features such queues, spill overs and congestion. The PARAMICS is a micro-traffic simulation software widely used in simulation. Many simulation-based evacuation studies deployed PARAMICS for traffic simulation studies. In Church and Sexton (2002), the authors developed a simulation model using PARAMICS to simulate the evacuation of a high-risk area in Santa Barbara called Mission Canyon Neighborhood. The model is used to analyze the possible evacuation scenarios changing traffic control, number of vehicles per household, opening of alternate exit and critical links in the network. Although these may not be the optimal ones, they are validated strongly based on empirical observations. The simulation provides the lower bound on the actual evacuation time as evacuee behavior is not considered and demand and arrival time estimates are not accurate. But the simulation results providing the best scenario in terms of the evacuation time might be indicative of the necessary factors to keep in mind in an emergency situation. In another study, In Chen (2006), PARAMICS was employed to compare the efficiencies of staged and simultaneous evacuation studied under free-flow and congestion situation. The study was performed for different network topologies. They made some simplistic assumptions like availability of route choices, destination information that prevents it from immediate realization. The findings, which include staged evacuation during congestion, would guide the design of new models. The aggregation of areas into zones is a technique employed to gain computational efficiency, but with a compromise in the precision. The default rules of PARAMICS for trip generation and destination and route choices were used for the simulation. They also provided a review of agent based simulation models and explained its advantage. They performed the simulation over three types of networks namely ring, grid and an actual road network. The study indicated that staged evacuations based on zones are more efficient than simultaneous evacuation. The models assume that the drivers are assumed to take the shortest path, which leads to frequent congestion, and hence the results may not reflect the best simultaneous evacuation time. CORSIM is another micro-simulation software used for traffic simulation. CORSIM is a combination of two other microsimulation models namely NETSIM and FRESIM, which are used for traffic simulation in surface streets and freeways respectively. In Sisiopiku et al. (2004), CORSIM was used to create a transportation model for Birmingham. The generated model was then used to test several emergency scenarios. They also made a brief review of micro-simulated evacuation models and discussed the vital features of CORSIM. Several response actions such as traffic diversions, altering signal timings, roadway clearance and access restriction were incorporated in the testing. The simulation model was then tested on different scenarios, namely: a discrete traffic event, evacuation situation and simulation of available response plans. The performance measures, which includes speed, queuing length and queuing time, were not justified to be the best set, and they do not guarantee that the simulated results correspond to the most efficient solution corresponding to the assumptions made. The extensibility of the model is a concern for networks besides the networks examined under the same settings. The model will not have immediate realizability, but is useful in validating and calibrating models and the recommendations with respect the measures that were focussed in the study. CORSIM was also used to test the efficiency of contra flows or lane reversal under various evacuation scenarios Williams et al. (2007). The study was carried out in very plain settings, just to test the effectiveness of lane reversal of the link in a network. The study could serve only as a guide to evaluate the lane reversal efficiency rather than stand alone model to establish evacuation routes. The simplicity of the model makes it computationally efficient to obtain quick results. We discuss the demerits of establishing lane reversal strategies using simulation techniques in section . Another popular micro-traffic simulation software, VISSIM, is also used in evacuation studies. In Yuan and Han (2005), VISSIM was used for nuclear power plant accidents with dynamic traffic assignment and most desirable destinations. In Tagliaferri (2005), a comparison was provided of VISSIM and CORSIM with the demand estimates provided by Federal Emergency Management Agency (FEMA) to study the efficiency of lane reversals. Both these models indicated increased throughput and decreased queue with lane reversal implementation. Throughput at key nodes, queue lengths and average speeds were considered to be the efficiency measures of the evacuation plan. These efficiency measures were compared between a lane reversal plan and a do-nothing plan. The evacuation scenarios were considered essentially for a reversing only two vital links of the network. The lane reversals were considered for only the critical links of the network, but the rest of the model is heavily dependent on the simulators used by the model. As microscopic simulations keeps track of individual entities in the system it is computationally expensive and it was often used in small scale evacuation systems. Also, the details kept by these models makes them more complex. Their function logic gets complicated for operation and they often result in befuddling number of parameters that are tough to keep track. However, this limitation is being slowly subdued with the advent of faster computers.

Macroscopic Simulation Techniques

As mentioned earlier macroscopic models captures the vehicles and their activities more coarsely compared to its counterpart. Traffic is aggregated and the aggregated flow will be studied through model variables such as speed, density and flow rate. NETVAC Sheffi et al. (1980) and MASSVAC Radwan et al. (1985) are two popular macro-traffic simulation software. NETVAC models radial evacuation from risk area, similar to evacuation during a nuclear accident. The travel demand is incidental in the model, since all the households within the risk area require evacuation. MASSVAC is a macroscopic simulation model developed for rural networks and was tested on a small rural network in Virginia for several loading curves. It is comprised of a community module to define the boundary of the hazard, a population characteristics module to determine the population allocation spatially and an evacuation module that performs the actual traffic assignment. It has a simple input structure and trip distribution. Later versions of MASSVAC have been developed to incorporate complex features such as user equilibrium assignments. Oak Ridge Evacuation Modeling System (OREMS) is a popular macro-simulation evacuation simulator. Identifying bottlenecks and feasibility of evacuation, establishing evacuation routes and identifying alternate conrod strategies are some of the important features of OREMS. GIS interface, which is a useful add-on in the recent years, is also available in OREMS. Moriarty et al., compared major macro-simulation softwares including OREMS, DYNEV and ETIS where the factors influencing the evacuation response were identified and several enhancements were suggested to improve the evacuation efficiency Moriarty et al. (2007). Most of these models were provided with the evacuation demand, as an input to the model and the models are a coarse approximation of reality. The increase in computational capacity in the recent years are making microsimulators more attractive, as they could also provide precision in performing localized studies and networks of reasonable sizes. However, macrosimulators could be integrated with microsimulators or analytical models. They could serve as quick validators in evaluating a developed model. Also, the very large scale network are still a little far fetched for analytical models and microsimulators and are mostly handled by macrosimulators. Macroscopic models are much simpler compared to the microscopic models, when it comes to to calibration and computation. This is only achieved through some shortcomings. They cannot be applied to instances that require individual vehicular dynamics to be tracked. As the vehicle positions are not known in these models, the modern traffic control systems such as ramp metering, lane change maneuvers and a few other features of intelligent transportation systems cannot be captured by these models. Also, evacuation that requires modeling of human behavior cannot employ macroscopic simulation studies for the same reason. The accuracy is compromised by the aggregation of the traffic and network. They are more applicable in large scale evacuation with respect to time and space, whenever these shortcomings are less realized.

Meso-Simulation Techniques

A new generation of simulators called meso-simulators are popular in the recent days. They combine the pros and cons of the micro and macro simulators. The meso-simulators clubs the vehicles into packets or platoons, which are then simulated as separate entities. In de Silva et al. (2003), CEMPS, a meso-simulator, was employed to develop a spacial decision support system. This is very practical model in the sense that the decision support system integrates the real time data obtained from geographical information system (GIS) to make traffic decisions and a object oriented simulation comprises of decision modeling and dynamic analysis components. CEMPS provides a convenient framework that permits this communication. The model does not sufficiently clarify the computational and economic concerns that may arise due to the increase in the size of evacuation network. In Gomes et al. (2001), the authors provided a meso-micro scale simulation study using SmartCAP that allows monitoring and studying of aggregate traffic flow behavior such as density, flow, and velocity and integrated it with a micro-simulator, SmartAHS, which is used to record the individual details. The integrated simulator consists of “window” of the micro-simulator that communicates with the meso-simulator. Essentially the vehicles are micro-simulated and the meso-traffic flow characteristics such as velocity, flow rate, etc. are preserved. The claimed boost in the computational speed is pertinent to specific situations by the use of meso-simulators. More precisely, the gain will be linear in terms of the level of aggregation of vehicles into packets. This still cannot justify the phasing out of macrosimulators or microsimulators. However, the level of aggregation will help in achieving the capability of a microsimulator to the desired accuracy. DYNEMO Payne (1971) proposed another mesoscopic traffic flow model that was developed, where unit of a traffic flow is an individual vehicle unlike packets. The motion of the vehicles is determined by their link's traffic density. The function that gives the relation between traffic density and speed is provided as an input to the model.

Integrated Techniques

Simulation techniques could be used in conjunction with analytical models in order to gain the best out of the optimization and simulation techniques. The evacuation plans that are generated through the optimization techniques could be a good lower bound on actual evacuation time, thus these plans could serve as a candidate for simulators to identify the discrepancies in the model, foresee the requirements and capture the features that were not incorporated in the optimization technique. In Sbayti and Mahmassani (2006), the authors provided a bi-level, simulation-based approach to solve the evacuation problem. At one level, the time-dependent route assignments are determined, and at another level a dynamic loading problem is solved and the output is later aggregated. The time-dependent route assignment is solved using the method of successive averages, and the traffic-demand simulation is done using DYNASMART-P to estimate the vehicle trip times and link travel times, thus achieving a system optimal schedule. The dynamic traffic assignment is implicity handled by DYNASMART-P, relieving the optimization model. The model is computationally efficient, but the gain in speed could be softened by the degree of accuracy of the heuristic with the increase in problem size and the complexity due to new features when extended to other situations. In Zou et al. (2005), an evacuation system was presented for ocean city integrating optimization and simulation techniques. An evacuation plan is generated by the optimization module and the plan is revised by the results of simulation evaluation. The pros and cons of the plans established were discussed. The model provides route choice options that permits users to deviate from the available or pretested plans, but the model assumes the availability of preestimated or forecasted demands. The formulation is based on cell transmission model proposed in Daganzo (1994) with necessary flow conservation and demand constraints. The result of this technique is a set of candidate plans, which could later be finalized using calibrated simulators. The results were then evaluated with a microscopic simulation program. This cell-based formulation makes the model more complicated in terms of the size and complexity, when dealt over a larger system, and the number of plans that have to be stored and tested in the next phase. The method is comprehensive compared to an integrated technique that produces one single solution, but makes it computationally unattractive. The candidate set could be a limited set to overcome this undesired effect. A similar two-level optimization method for evacuation of the ocean city was proposed in Liu et al. (2006b). At a broad level, they maximize the throughput at the higher level in terms of the number of vehicles being evacuated and minimize the travel time at the lower level. The cell-based formulation was made by accommodating cells of varying sizes in order to decrease the complexity of the optimization model. In Liu et al. (2005), another integrated technique was proposed in which the cell-based optimization model was used to formulate the demand constraints and the flow and storage capacity constraints. The result of the optimization module is then fed as the input to the microscopic simulator, CORSIM, which models real-time operational constraints and driver behavior that were not captured in the optimization model. The evacuation systems proposed were modeled with static demands. These integration techniques are very useful for a number of reasons. The evacuation plan could be validated through simulation and hence can be reliable. The computationally expensive task could be simulated and other operations could be optimized and thus they attempt to seek an optimal balance between precision and speed.

Reviewed Features

A survey in Gwynne et al. (1999) identified 22 evacuation models and assessed them by grouping them based on four different perspectives, namely enclosure; for example: the fineness or coarseness of a network, population perspective where actions are taken based on individuals or by groups, behavioral perspective based on how the occupants react to the environment and the nature of model applications. In Santos and Aguirre (2004), a survey of simulation models was presented for emergency evacuation. They made a classification based on modeling approach, namely flow-based, agent-based and cellular automata and discussed their relative advantages from the perspective of evacuee behavior. A flow-based modeling comprises of nodes representing the structures or places to and from the evacuees have to be moved and arcs mapping to the hallways, roads, etc., that links these nodes. Since the individual characteristics are not emphasized in this modeling, the practical realization of these models is impeded. However, if these models were used in conjunction with analytical models, they could be very powerful tool in terms of both speed and precision. In cellular automata the evacuation space is discretized into cells or grids with specific capacity and the entities flow from one cell to another. The models based on cell-based transmission could be very accurate. In Algers et al. (1997), 32 different micro-simulation models were analyzed and compared the features available in the models. Scale of application, i.e. the size of the network that the model can handle, was one of the features that were discussed in the papers. A detailed statistics of the objects and phenomenon that the models included was provided. Queue spill backs, weaving, incidents and commercial vehicles were by and large modeled. Indicators of objectives showed that the speed, travel time, congestion and queue length were widely adopted by the models and indicators such as comfort and performance were rarely used. Analysis of telematic functions and interface were also discussed. A comprehensive survey on simulation studies was provided Radwan et al. (2005) and analyzed various simulation software that are available and presented all the components considered for evacuation by these software. The categories of evaluation of the models were modeling, behavior, operations and hazards. Most of these features have already been discussed. This analysis provided a generalized framework for simulation studies of emergency evacuation.

Lane Reversals and Traffic Control

Contra flows or lane reversals are being considered in the simulation models just like in optimization models. The recent trend and studies have indicated that there is a significant decrease in evacuation time with the implementation of lane reversals. A more efficient way of controlling the traffic is by employing traffic personnel. In Williams et al. (2007), traffic control was studied by capturing these scenarios and simulating them. The travel time was significantly less for the instances with lane reversal. In Theodoulou and Wolshon (2004), the authors undertook a research to study the effect of contra flow implementations in evacuation in New Orleans. They employed CORSIM to simulate the freeway configuration and two other scenarios with contra flow implementation. The study carried out various loading configurations of major highway permitting contra flows. The experiments were used to demonstrate the benefits of contra flow. A significant improvement in the measures of effectiveness, namely travel time and average speed, was achieved for the configurations permitting contra flow. In most cases, the contra flows assume complete lane reversals, that is, the entire capacity of the road is switched toward the destinations. There are a couple of reasons for holding a capacity in either direction. In case of a large network, a link of the networks in either direction could be used in a route to reach the destination. In case of a link failure, where alternate paths are required, the reversed links can no longer be part of an alternate path to the destination. If this was not the case and even if a link cannot be a part of route from origin to destination, these arc capacities could be used in a routing problem where empty buses have to be routed back to pick-up points. Most of the papers currently assume that there will be enough number of buses to support the evacuation by making just one trip. Thus studies that identify the amount of capacities that has to be reversed within highway rather than complete reversals are limited. Simulation studies are not the best way to study the effects of contra flow as they need an input plan, which could be calibrated or validated, but cannot be a tool to identify the links or the amount of capacities that has to be reversed. Analytical tools could be employed for this purpose and the plans could later be tested and verified in simulations.

Dynamic Demand Estimation

In a large-scale evacuation, the evacuation distance is large and assuming static traffic conditions are not precise. DTA allows different traffic conditions to be included in the simulation. It captures the complex dynamic demand pattern that arises due to congestion, queues, spill back and delays. There will be a significant difference in the evacuation efficiency between static and dynamic demand. The factors that results in the difference has been discussed before. The dynamic demand estimation through a simulator may not be as difficult as the contra flows. In Antoniou et al. (1997), the authors provided a simulation model with a demand predictive capabilities implemented as a part of DynaMIT simulator. The model generates a pre-trip demand based on the historical demand and two systematic deviations based on daily demand fluctuations and driver response behavior. The pre-trip demand is used to establish a disaggregate pre-trip travel decisions for the drivers. Then a behavioral model is employed to exploit the available real time data and alter route decisions. Then a disaggregate origin destination matrix is generated based on the traffic count on the links of the network, which is then aggregated to estimate the new demand. The model takes into account various factors including behavior pattern, daily demand fluctuations and a random error component, thus giving more reliability to the model. The model employs a time discretization for determining intervals between which the distance updates are made. This makes the model computationally expensive based on the length of the interval. The question of consideration will be the capability of the model to handle other features in addition to the dynamic demand estimation. This model was just an extension of the work in Ashok and Ben-Akiva (1993). The demand estimation through update of historical demand based on driver behavior and was further enhanced by systematic deviations in the newer models. A similar method, QUEENSOD, Aerde et al. (2003) involves a seed matrix similar to the pre-trip demand matrix based on historical demand. The seed matrix establishes traffic counts and a micro-simulator is used to establish the traffic routes based on the estimate and then the seed matrix is altered to reduce the error between the estimated and observed traffic counts.

Miscellaneous Factors

The number vehicles per household in micro-traffic simulation may be important as this increases the demand. The use of critical links is a major factor in evacuation as it is important to ensure that the critical links are not congested, which would result in heavy traffic disruption. Incident management is a minor feature that models focuses on the need for alternate routes in case of accidents and estimation of traffic personnel for diversion and lane control. In Mitchell and Radwan (2006), the impact of staged evacuation efficiency was studied, where they considered six different scenarios and and they carried out simulation on a representative traffic network that resulted in successful staged evacuations. The scenarios comprised of combinations of shifting the departure times of evacuations. This resulted in increase in the total number of trips, but prevented congestion and queues. They concluded that the departure time shifting and total number of trips made had a positive impact on the clearance time.

Network Flow Problems with Lane Reversals

We study contraflow network problems, wherein we try to maximize flow in a graph while permitting direction reversals of an arc, resulting in a capacity increase in the direction of switch. The applications are realized in an emergency situation, where people have to be 'evacuated' from a specific area; *i.e.* a football stadium after a game, a city expecting a flood or hurricane, a zone where an unexploded ordnance device has been found, or a region that has been attacked by terrorists. In most of these cases, the evacuees are expected to leave the area of risk, the source(s), toward a safer place, the sink(s). A flow toward the source is undesired during most of these scenarios and we do not expect the evacuees to go in this direction. As a direct consequence, all the arcs that are not a part of any path from the source node(s) to the sink(s) might be left unused. One can even encounter idle arcs during certain scenarios, such as managing a football event, wherein we do have some amount flow toward the source. These idle arcs could be used to increase the efficiency of evacuation by reversing their directions. The scenarios involving in partial lane reversal capability could be captured with appropriate graph transformation. We discuss several scenarios that may arise during the reconfiguration, which includes permitting only a subset of arcs to be reversed, imposing a switching cost to the arcs involved in the reversals.

There are very few optimization techniques in the literature handling arc reversals. Kim and Shekar Kim and Shekhar (2005) proposed a simulated annealing procedure for this problem and provided empirical results. They also provide a sketch of the proof that the problem is NP-complete. A tabu-based heuristic was proposed by Tuydes and Ziliaskopolos Tuydes and Ziliaskopoulos (2006b) for the problem. They focus their study on a specialized version, where they permit lane reversals with partial capacities. Hamza-Lup et al. Hamza-Lup et al. (2004) proposed a heuristic for this contraflow problem. These techniques and their pitfalls were discussed in Kim and Shekhar (2005). A few other studies in the literature that are not analytical in nature were also proposed. They rely on simulation-based methods and decision support tools Theodoulou and Wolshon (2004); Williams et al. (2007).

In this chapter, we provide a detailed study of the arc reversal (or contra flow) problems with respect to their computational complexity. The motivation is to introduce the problems formally to provide a basis for further research in this area. As the applications are mainly realized during emergency situations, the dynamic flow problems are of principal interest, but we study static cases as presuppositions and also for the sake of completeness of the study. We provide a brief background of the network flow problems and explain the terminology used in the rest of the chapter. We then provide a discussion of static flow problems. A polynomial time algorithm through a graph transformation is introduced for the static maximum flow problem with arc reversal capability. The result is evident and it is useful in showing that the dynamic maximum contraflow problem, with single source and single sink, is polynomially solvable. We show that the the decision version of the multiple sources and multiple sinks version of the problem is NP-complete through a reduction from 3-SATISFIABILITY (3SAT). We show that the problem becomes NP-complete by having just two sources or sinks. In addition, we discuss the inability of the graph transformation that was employed earlier to provide feasible solutions. We finally show that the problem of finding the minimum total cost, incurred due to an arc switching cost, to identify the arcs to be reversed is NP-hard, even in the static case.

Background

The basic terminologies and definitions that are predominantly used in the network flows literature and that are essential for the rest of the chapter are explained in this section.

Definition 1 (Static feasible flow)**.**

Given is a graph $G = (V, A)$ with capacities $c_e \in \mathbb{Z}^+$ for all arcs $e \in A$. A static flow, characterized by the function $f : A \to \mathbb{R}^+$, with value v, from $s \in V$ to $t \in V$ is feasible, if

$$f_e \le c_e, \quad \forall e \in A \tag{21}$$

$$\sum_{(i,j)\in A} f_{i,j} - \sum_{(j,k)\in A} f_{j,k} = \begin{cases} v, & j = s \\ 0, & \forall j \in V \backslash \{s \cup t\} \\ -v, & j = t \end{cases} \tag{22}$$

We call node s as the 'source', node t as the 'sink' and rest of the nodes as 'intermediate' or 'transhipment' nodes.

Equation 21 ensures that the flow f_e along each arc $e \in A$ meets the capacity constraints; as we assume all lower bounds on the flow to be 0. In equation 22, the net flow out of s is v and t is $-v$. For all intermediate nodes it is 0 and is also referred to as flow conservation. The definition of a feasible flow generalizes in a natural way for the case of multiple sources and multiple sinks.

A sequence of distinct nodes $x_1, x_2, \ldots, x_n$ of a graph $G = (V, A)$ is called a chain if $(x_i, x_{i+1}) \in A, \forall i = 1, \ldots, n$. A chain is also referred to as a directed path. Let P be the set of all chains from s to t. We define another flow function, $h : P \to \mathbb{R}^+$, in terms of the flow along the chains from s to t. A feasible flow f with value v could be decomposed into a set of chains, P, from s to t, such that

$$v = \sum_{i=1}^{|P|} h_i \quad .$$

The process of obtaining flow along the chains this way is called as 'chain decomposition.' A more detailed account of these terminologies could be found in Ahuja et al. (1993); Ford and Fulkerson (1962).

In a dynamic graph or network $G = (V, A)$ each arc is associated with a travel time, $t : A \to \mathbb{R}^+$, besides the capacity function. The graph expanded over T time periods, $G^T = (V^T, A^T)$, is obtained by replacing each node by T copies and having nodes v_i^l and $v_j^{l+t_{i,j}}$ connected in $G^T = (V^T, A^T)$ if v_i and v_j are connected in G, for all $l = 0, \ldots, T - t_{i,j}$. This concept of a feasible flow can be directly adopted to the dynamic case by ensuring that both equations 21 and 22 are satisfied for all discrete time steps. Hence, a feasible dynamic flow is a feasible flow in the time expanded graph with the value equal to the sum of the net flows out of all the T copies of s. For more details about time expanded graphs refer, for instance, to (Ahuja et al., 1993, Chapter 19.6).

Maximum Static Contraflow Problems

In this section, we provide a polynomial time algorithm solving the maximum contraflow problem in a static graph. The results presented in this section are very basic and straightforward. Nevertheless, we discuss them in detail as this helps us in developing the main results later.

Now, let us define the maximum flow problem with arc reversal capability.

Definition 2 (Maximum Contraflow (MCF)).
***Instance**: Given a directed graph $G = (V, A)$ with source $s^+ \in V$, sink $s^- \in V$ and capacity $c_e \in \mathbb{Z}^+$ on each arc $e \in A$.*
***Question**: What is the maximum flow from node s^+ to node s^- if the direction of the arcs can be reversed?*

This problem is also called *maximum flow problem with arc reversal*. Consider now procedure P-MCF. In the first step, an auxiliary graph $\widetilde{G} = (V, \widetilde{A})$ is constructed. The transformation from the original graph G is obtained by summing the capacities of arcs (i, j) and (j, i). This allows us to reduce the MCF problem to the maximum flow problem on the transformed graph in step 2. Step 3 removes cycle flows in the transformed graph. This ensures that the constructed solution of the MCF problem in step 4 is well defined.

Procedure Maximum Contraflow (P-MCF)

1. Construct the transformed graph $\widetilde{G} = (V, \widetilde{A})$ where the arc set is defined as

$$(i, j) \in \widetilde{A}, \text{ if } (i, j) \in A \text{ or } (j, i) \in A \quad ,$$

   The arc capacity function $\widetilde{c}$ is given by

$$\widetilde{c}_{i,j} := c_{i,j} + c_{j,i} \quad ,$$

   for all arcs $(i, j) \in \widetilde{A}$.
2. Solve the maximum flow problem on graph $\widetilde{G}$ with capacity $\widetilde{c}$.
3. Perform flow decomposition into path and cycle flows of the maximum flow resulting from step 2. Remove the cycle flows.
4. Arc $(j, i) \in A$ is reversed, if and only if the flow along arc (i, j) is greater than $c_{i,j}$, or if there is a non-negative flow along arc $(i, j) \notin A$ and the resulting flow is the maximum flow with arc reversal for graph $G = (V, A)$.

End procedure

We have the prior knowledge that there exists an optimal flow to the maximum flow problem that does not have cycles. Thus, arcs on either direction will never be used in this flow for the maximum flow problem. This is the basic idea of procedure P-MCF that motivates the graph transformation given.

Theorem 1 (Proof of correctness)**.** *Procedure P-MCF solves the maximum flow problem with arc reversal for graph $G = (V, A)$ optimally.*

Proof. The proof consists out of two steps. First, we show that any solution of the procedure P-MCF is feasible for $G = (V, A)$. Second, we show its optimality.

For feasibility, we only have to show that step 4 in the algorithm is well defined; *i.e.* not both arcs (i, j) and (j, i) have to be switched. However, this is ensured by step 3. The optimal solution after the flow decomposition results in a set of paths from source to sink and a set of cycles with positive flows. After the flow decomposition we could cancel the positive flows along all cycles and ensure that there is no flow along any cycle. This ensures that there is either a flow along arc (i, j) or (j, i), but never on both arcs. Hence, the resulting flow from step 4 is a feasible flow with arc reversal for graph $G = (V, A)$.

Now, we prove that the resulting flow is also optimal. Note that any optimal solution to the maximum flow problem with arc reversal on graph $G = (V, A)$ is also a feasible solution to the maximum flow problem on the transformed graph $\widetilde{G} = (V, \widetilde{A})$. As the amount of flow send from s to t is not changed in steps 3 and 4, the resulting flow is an optimal solution to the maximum flow problem with arc reversal on graph $G = (V, A)$. □

The running time of procedure P-MCF is dominated by solving a maximum flow problem in step 2 and by the flow decomposition in step 3; as steps 1 and 4 can be done in $O(|A|)$. Let us denote the running time for solving the maximum flow problem by $S_1(|V|, |A|)$ and for the flow decomposition problem by $S_2(|V|, |A|)$. Then, the running time of procedure P-MCF is given by $O(S_1(|V|, |A|) + S_2(|V|, |A|))$. Using the highest-label preflow-push algorithm leads to $S_1(|V|,|A|) = O(|V|^2 \cdot \sqrt{E})$, Cheriyan and Maheshwari (1989). The flow decomposition can be done, for instance, in $O(|V| \cdot |E|)$, Ahuja et al. (1993). This proves the following theorem.

Theorem 2 (Running time)**.** *Procedure P-MCF solves the maximum contraflow problem in strongly polynomial time.*

We are now able to extend the result above to the case of multiple sources and multiple sinks. This problem is also called *maximum transshipment contraflow (MTCF) problem.*

Corollary 1. *The static version of the maximum contraflow problem with multiple sources and multiple sinks is polynomially solvable.*

Corollary 1 can be realized through a simple reduction. Let S^+ and S^- be the set of sources and the set of sinks, respectively. Then, add a 'super-source' u^+ and a 'super-sink' v^- together with the arcs (u^+, s^+), for all $s^+ \in S$, with arc capacities equal their respective surplus and (s^-, v^-) for all $s^- \in S^-$ with their arc capacities equal their respective deficits. For more details, refer to Ford and Fulkerson (1962).

Recognize that we basically show in this section that the maximum contraflow problem is equivalent to a maximum flow problem on an undirected (modified) graph. This could be seen in the graph transformation provided in step 1 of procedure P-MCF with arcs having same capacities in either directions.

Maximum Dynamic Contraflow Problems

In this section, we discuss the *maximum dynamic contraflow (MDCF) problem.* The maximum dynamic flow problem was studied by Ford and Fulkerson Ford and Fulkerson (1962), where they try to maximize the flow sent from source to sink, within a given time horizon T. Unlike the static case, in the dynamic network flow problem the flow over an arc can be repeated over time. Ford and Fulkerson proved that this problem is equivalent to solving a minimum cost flow problem with the arc costs as travel times on the arcs. Then the optimal flow on the arcs from source to sink is decomposed into a set of paths or chains. These chains are then *temporally repeated* over time to obtain the required dynamic flow. In other words, there is always a temporally repeated chain flow that is equivalent to the maximum dynamic flow. Let us assume there are P paths obtained from the chain decomposition of the optimal minimum cost flow. Then the maximum dynamic flow is given by

$$\sum_{i \in P} (T + 1 - t_i) h_i \quad ,$$

where h_i is the flow along the i^{th} path and t_i is the time taken to travel the i^{th} path. In this section, we first study the single source and single sink dynamic flow problem having arc reversal capability. We provide an algorithm employing a similar kind of graph transformation as procedure P-MCF and discuss its proof of correctness together with its worst case running time analysis. This implies that the *quickest contraflow (QCF) problem* is also polynomially solvable. In the *quickest flow problem*, the time to send a given flow from source to sink is minimized. Burkard et al. Burkard et al. (1993b) gave a strongly polynomial time algorithm for this problem.

Hoppe Hoppe (1995) studied the multiple sources and multiple sinks version of this problem, also called the *quickest transshipment problem*, where they minimize the time taken to send the supply at the sources to the sinks satisfying their demands. In static network flows, the multiple sources and multiple sinks are handled by adding a 'super-source' and a 'super-sink.' Then they are connected to the sources and sinks respectively, see Corollary 1. However, this solution procedure is not applicable in a dynamic case anymore. For the same reason, the dynamic contraflow problem with multiple sources and multiple sinks is NP-complete. We provide an example illustrating this together with a proof of its NP-completeness.

Single Source and Single Sink

Let us extend the MCF problem to the dynamic case.

Definition 3 (Maximum Dynamic Contraflow (MDCF))**.**
INSTANCE: *Given a directed graph $G = (V, A)$ with source node $s^+ \in V$, sink node $s^- \in V$, capacity $c_e \in \mathbb{Z}^+$ and transmission time $t_e \in \mathbb{Z}^+$ on each arc $e \in A$ with $t_{i,j} = t_{j,i}$ if $(i, j), (j, i) \in A$, and an overall time horizon $T \in \mathbb{Z}^+$.*
QUESTION: *Determine the maximum amount of flow that can be send in T units of time from source s^+ to sink s^-, if the direction of the arcs can be reversed at time 0.*

Note: In this case, if we choose to switch an arc, it remains switched from time 0 to T. The case where we allow switching of arcs back and forth in time is trivial as the quickest transhipment contraflow problem, with this assumption, reduces to the quickest transhipment problem through the graph transformation suggested in procedure P-MDCF and hence is polynomially solvable.

Definition 3 states that in a MDCF problem, the graph is allowed to be asymmetric with respect to the arc capacities. However, whenever both directions of an arc are included in the graph, then the traveling time of these two arcs must be the same. This assumption implies that the switching of an arc only changes the capacities of the arcs but does not alter their traveling time.

The concept of temporally repeated flows is very fundamental for the maximum flow problem with single source and single sink. Our algorithm for solving the MDCF problem is mainly based on this concept. Hence, let us repeat the definition given by Ford and Fulkerson, (Ford and Fulkerson, 1962, page 147).

Definition 4 (temporally repeated)**.**
A dynamic flow that can be generated by repeating chain flows of a static flow in graph G is called temporally repeated flow.

The following theorem reveals the usefulness of temporally repeated flows in the context of single source and single sink network flow problems, (Ford and Fulkerson, 1962, Theorem 9.1).

Theorem 3. *There is a temporally repeated dynamic flow that is maximal over all dynamic flows for T periods.*

The flow to be temporally repeated could then be determined by just solving a minimum cost flow problem. Let us denote its running time by $S_3(|V|, |A|)$. Using, for instance, the minimum mean cycle-canceling algorithm leads to a strongly polynomial running time of $O(|V|^2 \cdot |E|^3 \cdot \log(|V|))$, Goldberg and Tarjan (1989).

Before we proceed to the next lemma, we need to know that utilizing the concept of time expanded graphs in a solution algorithm leads to a pseudo-polynomial running time. In this case, the running time depends on $|T|$, rather than $\log(|T|)$ which would then lead to a weakly polynomial running time. Nevertheless, we use the concept of time expanded graphs in Theorem 4.

Consider now procedure P-MDCF. We show in Theorem 4 that it solves the MDCF problem correctly. The main differences of procedure P-MCF and P-MDCF is given in step 2. For the dynamic problem, we need temporally repeated flows. This ensures that only one of the arcs (i, j) or (j, i) is used in the flow. This enables us to use the same flipping rule for the arcs as in procedure P-MCF.

In order to show the correctness of procedure P-MDCF, we need the following lemma.

Procedure Maximum Dynamic Contraflow (P-MDCF)

1. Construct the transformed graph $\widetilde{G} = (V, \widetilde{A})$ where the arc set is defined as

$$(i,j) \in \widetilde{A}, \text{ if } (i,j) \in A \text{ or } (j,i) \in A \quad .$$

   The arc capacity function $\widetilde{c}$ is given by

$$\widetilde{c}_{i,j} := c_{i,j} + c_{j,i}$$

   and the traveling time is

$$\widetilde{t}_{i,j}\left(= \widetilde{t}_{j,i}\right) := \begin{cases} t_{i,j}, & \text{if } (i,j) \in A \\ t_{j,i}, & \text{otherwise} \end{cases} \quad ,$$

   for all arcs $(i,j) \in \widetilde{A}$.

2. Generate a dynamic, temporally repeated flow on graph $\widetilde{G}$ with capacity $\widetilde{c}$ and traveling time $\widetilde{t}$.
3. Perform flow decomposition into path and cycle flows of the flow resulting from step 2. Remove the cycle flows.
4. Arc $(j,i) \in A$ is reversed, if and only if the flow along arc (i,j) is greater than $c_{i,j}$, or if there is a non-negative flow along arc $(i,j) \notin A$ and the resulting flow is the maximum flow with arc reversal for graph $G = (V, A)$.

End procedure

Lemma 1. *The maximum amount of flow in the single source and single sink maximum dynamic contraflow problem for graph $G = (V, A)$ is less than the optimal flow in the maximum contraflow problem for the corresponding time expanded graph $G^T = (V^T, A^T)$.*

Proof. The result follows directly from the observation that every feasible flow to the maximum dynamic contraflow problem has an equivalent feasible flow to the maximum contraflow problem of the time expanded graph. □

Please note that Lemma 1 holds good for more than one source and one sink. However, in general, equality holds only for the case of a single source and a single sink, as we will see in the following theorem. We are now ready to prove the correctness of procedure P-MDCF.

Theorem 4 (Proof of correctness)**.** *Procedure P-MDCF solves the maximum dynamic contraflow problem for graph $G = (V, A)$ optimally.*

Proof. The concept of this proof is similar to the proof of Theorem 1. First, we prove that all the steps in procedure P-MDCF are well defined and result in a feasible solution. Second, we show optimality.

For feasibility, the proof follows directly from the fact that the constructed flows are temporally repeated and hence, there is only a flow in one direction of two nodes, and never in both directions at the same time as well as at different time periods. After canceling the flows along the cycles, we have flows either on arc (i,j) or on (j,i) but not on both. This ensures that the flow is less than the reversed capacities on all the arcs at all time units. This also ensures the feasibility. In other words, we now have established the fact

$$[G = (V, A)]_{MDCFopt} \geq [\widetilde{G} = (V, \widetilde{A})]_{MDFopt} \quad ,$$

by the argument that every feasible flow of the dynamic flow problem in the transformed graph $\widetilde{G} = (V, \widetilde{A})$ is feasible to the maximum dynamic contraflow problem in the graph $G = (V, A)$. Our proof is complete if we show that

$$[G = (V, A)]_{MDCFopt} \leq [\widetilde{G} = (V, \widetilde{A})]_{MDFopt} \quad .$$

To see this, first note that the maximum contraflow in graph $G^T = (V^T, A^T) \geq$ maximum dynamic contraflow in graph $G = (V, A)$, from Lemma 1. Hence we have,

$$[G = (V, A)]_{MDCFopt} \leq [G^T = (V^T, A^T)]_{MCFopt} \quad .$$

By Theorem 1 we have that the maximum contraflow problem in graph $G^T = (V^T, A^T)$ is equivalent to the maximum flow problem in the graph $\widetilde{G}^T = (V^T, \widetilde{A}^T)$, where the arc set $\widetilde{A}^T$ is defined as

$$(i, j) \in \widetilde{A}^T, \text{ if } (i, j) \in A^T \text{ or } (j, i) \in A^T \quad ,$$

and the arc capacity function $\widetilde{c}$ is given by

$$\widetilde{c}^t_{i,j} := c^t_{i,j} + c^t_{j,i} \quad .$$

Thus,

$$[G^T = (V^T, A^T)]_{MCFopt} = [\widetilde{G}^T = (V^T, \widetilde{A}^T)]_{MFopt} \quad .$$

By Theorem 3, the maximum flow in the time expanded graph $\widetilde{G}^T = (V^T, \widetilde{A}^T)$ can be obtained by a temporally repeating a chain flow of a static graph $\widetilde{G} = (V, \widetilde{A})$. Hence we have the fact,

$$[\widetilde{G}^T = (V^T, \widetilde{A}^T)]_{MFopt} = [\widetilde{G} = (V, \widetilde{A})]_{MDFopt} \quad .$$

□

Just like procedure P-MCF, running time dominating are steps are 2 and 3 for procedure P-MDCF. This results in a worst-case running time of $O(S_2(|V|, |A|) + S_3(|V|, |A|))$; which is strongly polynomial.

Theorem 5 (Running time)**.** *Procedure P-MDCF solves the maximum flow problem in strongly polynomial time.*

For given excess b, the *quickest-contraflow problem* determines the minimum time horizon T needed by any feasible flow.

Corollary 2. *The quickest-contraflow problem can be solved in a strongly polynomial time.*

One way to realize Corollary 2 is through the work by BURKARD et al. for the quickest flow problem, Burkard et al. (1993b). First, obtain an upper bound on the quickest time and second, perform a binary search by repeatedly solving the minimum dynamic contraflow problem. Such a bound can be obtained in polynomial time, for instance, by computing a path from source to sink and temporally repeating flow along the path until all supply at the source is sent to the sink. However, this leads to a weakly polynomial algorithm. A strongly polynomial algorithm could be obtained through a parametric search suggested by MEGIDDO Megiddo (1979); Burkard et al. (1993b).

Multiple Sources and Multiple Sinks

Let us start with the definition of the multiple sources and multiple sinks version of the MDCF problem.

Definition 5 (Dynamic Transshipment Contraflow (DTCF))**.**
INSTANCE: *A directed graph $G = (V, A)$, a set of sources $S^+ \subset V$, a set of sinks $S^- \subset V$, arc capacities $c_e \in \mathbb{Z}^+$ and transmission time $t_e \in \mathbb{Z}^+$ for each arc $e \in A$ with $t_{i,j} = t_{j,i}$ if $(i, j), (j, i) \in A$, and an overall positive integer time bound T.*
QUESTION: *Is there a feasible dynamic flow within time horizon T, allowing each arc to be revered once at time 0?*

Note that the DTCF problem is a decision problem corresponding to the maximum dynamic contraflow problem with multiple sources and multiple sinks.

DTCF is NP-complete in the strong sense

In this section, we prove that the DTCF problem is NP-complete. A sketch of the proof outline was given in Kim and Shekhar (2005). However, we provide a rigorous proof. Also, the proof has some differences though we provide the reduction from the same problem, *3SAT*, (Garey and Johnson, 1979, page 46):

Definition 6 (3SAT)**.**
INSTANCE: *Collection* $C = \{c_1, c_2, \ldots, c_m\}$ *of clauses on a finite set* U *of variables such that* $|c_1| = 3$ *for* $1 \leq i \leq m$.
QUESTION: *Is there a truth assignment for* U *that satisfies all the clauses in* C*?*

3SAT is known to be NP-complete in the strong sense, see (Garey and Johnson, 1979, Theorem 3.1).

For an instance of 3SAT, construct a graph $G_{3SAT} = (V, A)$ for DTCF as follows. For each clause c_i we have one source node c_i^+ with a surplus of 1. Each variable $u_j \in U$, is presented by six nodes in the graph: two for each literal, named u_j^1, u_j^2, $\overline{u}_j^1$ and $\overline{u}_j^1$ respectively, one source node with surplus 1, d_j^+, and one sink node with deficit -1, d_j^-. Finally, there is one node with deficit $-|C|$, named s^-. This sums up to $|V| = |C| + 6|U| + 1$ nodes. Each clause node c_i^+ is connected to the nodes with superscript 1 representing its literals, taking 3 time units. For each j, the node u_j^1 is connected to its copy, u_j^2, with transshipment time of 1. Nodes d_j^+ are connected to u_j^2 and $\overline{u}_j^2$ with transshipment time 1, while nodes d_j^- are connected to u_j^1 and $\overline{u}_j^1$ having a transshipment time of 1. Finally, each second copy (superscript 2) of the literals is connected to the sink s^- taking a time of 1. All arcs have a capacity of $|C|$. This leads to $|A| = 3|C| + 8|U|$ arcs in graph G_{3SAT}. One such graph transformation is shown in Fig. 1.

The proof of the validity of the transformation is based on the following key observation.

Lemma 2. *In any feasible flow* f *in the graph* G_{3SAT} *within time* $T = 5$*, there is a flow of value 1 from node* d_j^+ *to node* d_j^-*, for all* j.

Proof. Let us fix index j and assume that the flow to node d_j^+ is integral. If the flow to node d_j^- does not come from node d_j^+, then it can only come from exactly one of the nodes c_i or d_k^+ with $k \neq j$. However, in both cases, the flow arrives at node d_j^- earliest at time 6, or 7 respectively. This proofs the lemma for the case of integer flows. The case of fractional flow is similar: If some fraction of the flow to node d_j^- comes from a different node then d_j^+, then the flow arrives after time $T = 5$. □

Lemma 2 implies that for a feasible flow, at least one of the arcs (u_j^1, u_j^2) or $(\overline{u}_j^1, \overline{u}_j^2)$ has been switched for all j – with other words, at most one of the two arcs (u_j^1, u_j^2) and $(\overline{u}_j^1, \overline{u}_j^2)$ keep their direction in any feasible flow with time bound $T = 5$. Now, we are able to proof the following lemma.

Lemma 3. *An instance of 3SAT is a 'YES' instance, if only if the transformed graph* G_{3SAT} *is a 'YES' instance for DTCF with overall time bound* $T = 5$.

Proof. "⇒" Let 3SAT have the feasible assignment $u_j = a_j$ for all variables, with $a_j \in \{0, 1\}$. Then, reverse the arcs (u_j^1, u_j^2) if $a_j = 0$, and reverse arc $(\overline{u}_j^1, \overline{u}_j^2)$ otherwise. Now, for all j, send one unit of flow from d_j^+ to d_j^- along the reversed arc. As only one of the arcs (u_j^1, u_j^2) or $(\overline{u}_j^1, \overline{u}_j^2)$ has been switched, we can send flow from any of the nodes c_i^+ through any non-switched arc, dependent on the assignment of the literals. This leads to a feasible flow for DTCF within time $T = 5$.

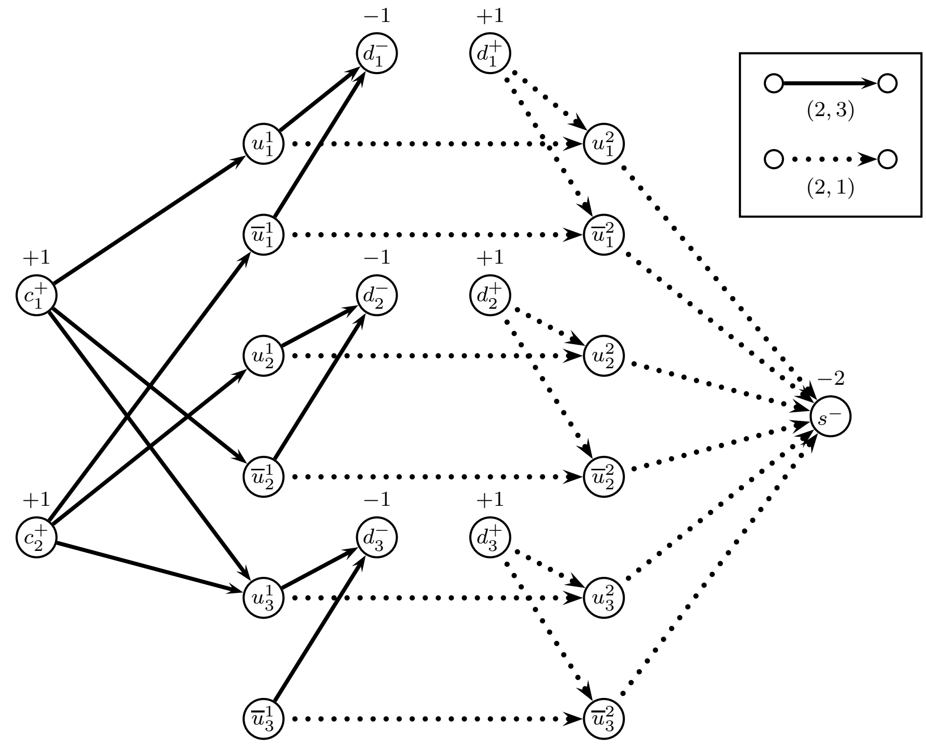


Figure 1: Transformed graph G_{3SAT} corresponding to 3SAT instance

"$\Leftarrow$" We have to show, that any feasible flow f for DTCF needing (at most) 5 units of time leads to a 'YES' instance of the 3SAT. We assign the following value to each variable $u_j \in U$ as

$$u_j := \begin{cases} 0, & \text{if arc } (u_j^1, u_j^2) \text{ is reversed in flow } f \\ 1, & \text{otherwise} \end{cases} . \tag{23}$$

We have to show that this is a satisfying truth assignment for the 3SAT instance. Now, assume that clause c_i is not a truth assignment. One unit of flow is send from node c_i^+ to node s^- through one of the nodes u_j^1 or $\overline{u}_j^1$ with $u_j \in c_i$ or $\overline{u}_j \in c_i$. Notice that this flow cannot go through any other node c_k^+ with $1 \leq k \leq m$ and $k \neq i$. Lemma 2 implies that the corresponding value of variable u_j has been set; *i.e.* $u_j = 1$ if the flow passes node u_j^1, or $\overline{u}_j = 1$ if it passes through node $\overline{u}_j = 1$. This leads to a contradiction. □

The second part of the proof of Lemma 3 together with Lemma 2 give the idea of the transformation from *3SAT*. First, we have to send one unit of flow from each of the nodes d_j^+ to d_j^-. This ensures that (at least) one of the arcs between the copies of the literals has to be reversed. The arc which has not been switched can then be used for the flow of the nodes c_j^+, allowing the clauses to have a truth assignment. Hence, the value of the literals is reflected by the switching of the arcs.

Theorem 6. *DTCF is* NP*-complete in the strong sense.*

Proof. DTCF $\in$ NP, as a non-deterministic algorithm needs only guess the set of arcs to be reversed together with a flow f and check if the flow is feasible with time bound $T = 5$; which can all be done in polynomial time. Lemma 3 states that the given transformation G_{3SAT} from *3SAT* to DTCF is valid. As the cardinality of the node set and the arc set of the constructed graph is $O(|C|)$, the transformation is polynomial in the input size of *3SAT*. $\square$

We want to mention that the transformation from *3SAT* can easily be changed to the general SAT by only changing the appropriate arcs from the clause nodes to the nodes representing the literals.

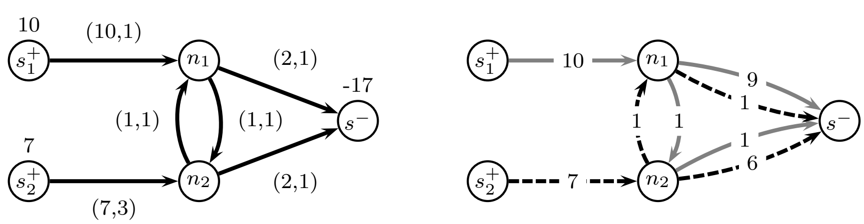
What makes DTCF so tough to solve?

Ford and Fulkerson introduced the idea of temporally repeated chain flows of a static flow. This enabled them to solve the maximum dynamic flow problem with one source and one sink. The fundamental principle is that there is always an optimal dynamic flow which uses only one direction of an arc, but never both. They call this a *standard chain decomposition*. This property allows us to solve the maximum dynamic flow problem in strongly polynomial time. We exploit this property to solve the MDCF problem.

The concept of standard chain decomposition is not sufficient for some well known dynamic flow problems Hoppe (1995); Hajek and Ogier (1984); Orlin (1983). An example is given in Fig. 2. Graph $G = (V, A)$ shown in Fig. 2 (a) has a feasible flow with time $T = 6$ as illustrated in Fig. 2 (b). The dashed and gray lines show the two flows from nodes s_1^+ and s_2^+ to node s^-, respectively. Analyzing the graph reveals that there is no feasible flow within time horizon $T = 6$ using only one of the arcs (n_1, n_2) or (n_2, n_1); this can be seen, for instance, by considering the flow trough the cut separating s^- from the rest of the graph.

However, it was still possible to solve the maximum dynamic flow problem with multiple sources and multiple sinks in the time-expanded graph; resulting in a pseudo-polynomial running time algorithm. However, Hoppe was able to provide a polynomial time algorithm for the dynamic transshipment problem Hoppe (1995). He introduced the concept of *non-standard chain decomposition*, allowing flow in either directions of an arc at different time steps – if both directions of an arc are present in the graph.

Loosely speaking, the procedures P-MCF and P-MDCF reverse the arcs on the fly and they are blind whether they reverse an arc or not. This does not cause any problems in the context of static flows or single source and single sink dynamic flows, as in a standard chain decomposition, one can always derive an optimal solution using only one the arcs during the whole time horizon. However, in the case of multiple sources and multiple sinks, the potential of using both arcs leads to the problem that we have to know if an arc has been reversed or not. But exactly this memory and the tradeoff of reversing the arc now or at a later time, makes the problem NP-complete. Consider Fig. 2 again. Applying the idea of procedures P-MDCF to this problem leads to the following result: At time 1, we would switch arc (n_2, n_1) in order to increase the capacity and at time point 3, we would switch it back again; resulting in a flow needing only $T = 5$ time steps.



(a) Graph $G = (V, A)$

(b) Feasible flow with $T = 6$ using both arcs (n_1, n_2) and (n_2, n_1)

Figure 2: A tough instance of DTCF

We showed that DTCF is NP-complete. The reduction from *3SAT* involves $|C| + |U|$ source nodes and $|U| + 1$ sink nodes. In the following, we show that there is no polynomial time algorithm for the DTCF problem having only two sources and one sink (or one source and two sinks), unless $\mathsf{P} = \mathsf{NP}$. In other words, allowing only one more source or sink to DTCF makes the problem NP-complete.

We do not go into full detail here, but rather provide the idea of a reduction from PARTITION, which is motivated by the key observation of Lemma 2 and the NP-completeness proof by MELKONIAN, Melkonian (2007). Given is a finite set A and a size $a_i \in \mathbb{Z}^+$ for each $i \in A$. The PARTITION problem decides whether there is a subset $\bar{A} \subseteq A$ such that $\sum_{i \in \bar{A}} a_i = \sum_{j \in \bar{A} \setminus A} a_j$, or not. PARTITION is known to be NP-complete (in the weak sense), see (Garey and Johnson, 1979, Theorem 3.5, Chapter 4.2). Let $\sum_{i \in A} a_i = 2L$ with $L \in \mathbb{Z}^+$. We construct an instance of the DTCF with two source nodes s_1^+, s_2^+ and one sink node s^-, as shown in Fig. 3. The idea of this transformation is that the flow at node s_2^+ has to pass through node v_0^1 to reach node s^-, and one unit of flow from node s_1^+ has to travel though node v_n^1 to node s^-. This is indeed true as otherwise the total time bound of $T = 2L + 2$ would be exceeded. The flow through the nodes v_0^1 to v_n^1 and back gives the assignment to set $\bar{A}$; *i.e.* $i \in \bar{A}$ if and only if arc $(v_{i_1}^1, v_i^1)$ is not reversed in the graph.

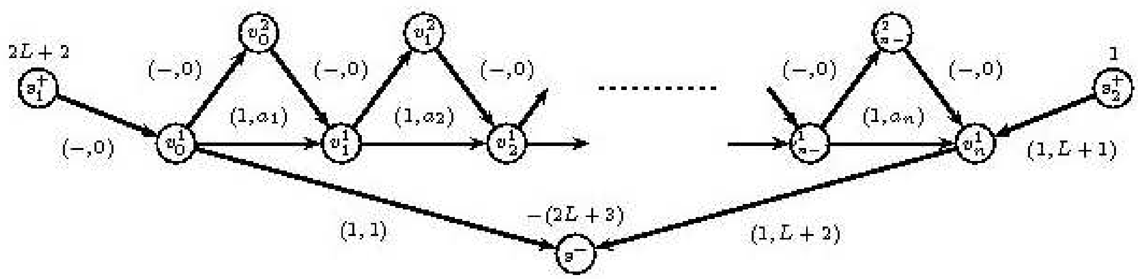


Figure 3: Instance for DTCF with time bound $T = 2L + 2$ resulting from PARTITION

Contraflow Problems with Arc Switching Cost

To allow the switching of an arc in order to increase the capacity in one direction results from the application in evacuation scenarios. However, in practice, you might not be able to switch certain arcs. For instance, in evacuation scenarios, certain streets are reserved for emergency vehicles but can also be used by (limited number of) other travelers; *i.e.* this can be modeled by reducing the capacity of this arc and blocking it from being reversed. In addition, the switching of an arc is highly costly; *i.e.* in order to switch the direction of a highway, we have to set up police blocks on each entry to the highway. Hence, it is natural to ask what are the minimum cost incurred in switching the arcs allowing a certain (minimum) amount of flow. This leads to the following problem.

Definition 7 (Fixed Switching Cost Contraflow (FSCF))**.**
Instance: *A directed graph $G = (V, A)$ with a set of sources S^+, a set of sinks S^-, excess $b \in \mathbb{Z}^{|V|}$, arc capacities c_e and arc-switching cost b_e^f for each arc $e \in A$.*
Question: *Find a feasible flow f in G with minimal total cost, if the direction of the arcs can be reversed with (fixed) cost b^f.*

Note that FSCF is a static problem with multiple sources and multiple sinks. The fixed cost b_e^f occur, whenever arc e is reversed. This definition allows to model the situation described above: Whenever an arc cannot be reversed, then its cost can be assigned a high value; *i.e.* Big M. As the cost of switching can differ for each arc, we can distinguish between the effort of reversing an arc; *i.e.* reversing a highway or an alleyway involves different cost or resources.

The fixed switching-cost contraflow problem has the following interesting value. One can solve the MTCF problem and determine the optimal flow in the graph, see Corollary 1. Later, one can apply the FSCF problem to determine the minimal cost implied by the switching of arcs, while still pushing the optimal amount of flow trough the graph.

Notice that the FSCF problem has a similar structure as the *minimum concave-cost network flow* problems. These problems ask to find a feasible flow while minimizing the total cost which are in this case the sum of concave-costs induced by using of the arcs. For an exact definition and an overview about this problem, please see the survey by Guisewite and Pardalos, Guisewite and Pardalos (1990). We can basically assume the concave-cost per arc to consist of fixed cost, occurring whenever this particular arc is used, and a variable cost, depending on how much flow is send trough this arc, see Kim and Pardalos (2000). Fixing the variable cost to zero leads to a special problem called *minimum cost fixed flow* (MCFF) problem. Krukme et al. prove that this problem is NP-hard in the strong sense even on series-parallel graphs, (Krumke et al., 1998, Theorem 14). Series-parallel graphs have a very special structure and are defined recursively, see Gross and Yellen (2003); Bern et al. (1987). Furthermore, Krukme et al. show that the minimum cost fixed flow problem is equivalent to the following problem, (Krumke et al., 1998, Theorem 8):

Definition 8 (0/1-Minimum Improvement Flow (MIF))**.**
Instance: *A graph $G = (V, A)$ with sink node s^+, source node s^-, excess $b \in \mathbb{Z}^{|V|}$, arc capacities $c_e \in \mathbb{Z}^+$, maximum capacities $C_e \in \mathbb{Z}^+, C_e \geq c_e$ and capacity improvement cost $\bar{b}_e \in \mathbb{Z}$.*
Question*: Determine an improvement strategy $d : A \to \{0, C_e - c_e\}$ with minimum cost $\sum_{e \in A} d_e \bar{b}_e$, such that the graph with the improved capacity $u_e + d_e, \forall e \in A$, allows a feasible flow f from s^+ to s^-.*

The definition given here is slightly different then the one in the paper by Krukme et al., (Krumke et al., 1998, Definition 7). Basically, we assume all data to be positive integral. The improvement strategy function d is a 0-1 decision if additional capacity is used or not; independent of how much additional capacity is used. The cost for this additional capacity for arc e is fix at value $(C_e - c_e)\bar{b}_e$. In order to prove that FSCF is strongly NP-hard, we show that it is equivalent to MIF.

Theorem 7. *Fixed switching-cost contraflow is equivalent to 0/1-minimum improvement flow.*

Proof. Without loss of generality, we can assume the FSCF problem to have single source and single sink. Recognize that the graph transformation provided for Corollary 1 works here.
"⇒" Given an instance of FSCF for graph $G = (V, A)$ with arc capacity c_e and arc-switching cost b^f_e. Construct an instance of MIF for graph $\mathbb{G} = (\mathbb{V}, \mathbb{A})$ as follows. If there is an arc $(i, j) \in A$ and $(j, i) \notin A$, then $(i, j), (j, i) \in \mathbb{A}$ with ${}_{i,j} = \mathbb{C}_{i,j} = \mathbb{C}_{j,i} := c_{i,j}, \bar{\ }_{i,j} = {}_{j,i} := 0$, and $\bar{\ }_{j,i} := b^f_{i,j}/c_{i,j}$ respectively. For the case that $(i, j), (j, i) \in A$, we define $(i, j), (j, i) \in \mathbb{A}$ with ${}_{i,j} := c_{i,j}$, $\mathbb{C}_{i,j} = \mathbb{C}_{j,i} := c_{i,j} + c_{j,i}, \bar{\ }_{i,j} := b^f_{j,i}/(c_{i,j} + c_{j,i})$, ${}_{j,i} := c_{j,i}$, and $\bar{\ }_{j,i} := b^f_{i,j}/(c_{i,j} + c_{j,i})$ respectively. By applying the cycle reduction principle we can see that this transformation is indeed valid.
"⇐" Given an instance of MIF for graph $G = (V, A)$ with c_e, C_e and $\bar{b}_e$, construct an instance of FSCF for graph $\mathbb{G} = (\mathbb{V}, \mathbb{A})$ as follows. For any arc $(i, j) \in A$, we have the three arcs $(i, j), (i, \bar{\imath}), (j, \bar{\imath}) \in \mathbb{A}$. Define ${}_{i,j} := c_{i,j}$, ${}^f_{i,j} = {}^f_{i,\bar{\imath}} := M$, ${}_{i,\bar{\imath}} = {}_{j,\bar{\imath}} = C_{i,j} - c_{i,j}$ and ${}^f_{j,\bar{\imath}} := \bar{b}_{i,j}(C_{i,j} - c_{i,j})$, where M is a big number preventing to switch the corresponding arc in an optimal solution. □

Recognize that having fixed cost for arc reversals makes the problem NP-hard, even in the static case. One reason is, for instance, the previously mentioned observation, that the procedure P-MCF is 'blind' for the arc reversal decisions. Adding a time component to FSCF makes it practically even more difficult to solve. The time component reveals also the differences between the (dynamic) fixed switching-cost contraflow problem and the (dynamic) 0/1-minimum improvement flow problem: MIF affects only a particular arc (i, j), while in FSCF also the reverse arc (j, i) is affected, if both arcs are contained in the graph.

A Branch-and-Price Mechanism for Multimodal Evacuation

Introduction

The survey on evacuation problems (Arulselvan et al. (2004)) indicates that there is a shortage of analytical techniques in multimodal evacuation studies. This chapter focuses on establishing efficient evacuation routes with bimodal transportation. We consider emergency management or event management situations such as football game, which assumes the absence of panic situations but still captures the several aspects of an evacuation settings. These include high demands during the event, need to satisfy demands quickly and congestion due to high demands. We assume private cars and buses as the modes of transportation. The cars are to take a path from source to destination, while the buses are routed. We assume that the routes of the buses are known. We need to establish efficient paths for the cars and determine the frequency of the buses along the routes. The problem is comparable to the line-planning problem Goossens et al. (2002); Abbas-Turki et al. (2004); Borndörfer et al. (2007), where multiple lines or modes of transportation are available and demands of people are available at specific time windows. The lines are predefined paths and the frequency of a line needs to be determined. Columns generation procedures are quite popular for the line planning problem and much research have been done in this area Borndörfer et al. (2007); Pfetsch and R.Borndrfer (2005). We employ the branch and price approach to solve the problem.

Multimodal Problem

Multimodal flow problems are known to be NP-hard Radwan et al. (2005). Thus, the problem, tailored to the needs of evacuation studies, requires efficient approaches to solve them either approximately or exactly. We provided a path-based formulation that would enable us to employ a branch and price procedure to solve the problem. We formally define the problem and state the assumptions.

Problem Definition

In this problem we have two sets of people depending on their modes of transport. We recognize the modes of transportation as private cars and buses. We know the demands of cars and people traveling by bus for every pair of node. We also have a set of bus tours that has already been established. The arcs of the network under consideration is shared by both cars and buses. Each link have a travel time and a capacity. In a time expanded network, each link has a cost depending on the time instance of the originating node and its distance from its nearest source node. We need to determine the most efficient path for the cars between the origins and destination and the frequency of the buses along their predetermined routes without exceeding the capacity of the arcs. The efficiency is determined based on the cost structure.

We formally define the problem as: Given a graph $G(V, E)$ with c_{ij}, u_{ij} and t_{ij} as the cost, capacity and travel time respectively on each arc $(ij) \in E$, a set of T bus tours, and two sets of origin destination pairs $(OD)_1$ and $(OD)_2$ corresponding to demands of origin and destinations of cars and people traveling by bus respectively, determine the minimum cost path of the cars and frequency of the bus routes satisfying demands and capacity.

Assumptions and realization

We make some simplistic assumptions, which does not hinder the realization of the model. We assume that the demands between the origin-destination pairs are known and remains static. We assume that the bus routes are established and we only need to determine their frequency. We also assume that the loading and unloading time of the buses is zero. The last assumption could, however, be overcome in the current model by appropriately changing the bus routes to accommodate fixed loading times. We now provide a path and route-based formulation that will enable us to implement a branch and price mechanism.

Formulation and discussion

A bimodal evacuation problem is considered in which two modes of evacuation, namely private cars and buses, are used for evacuating people from the origin nodes. The people have their destination preferences from the respective origins. In an evacuation setting, the buses are routed to pick up people from their origins and drop them at their destinations. The private cars are taken by people directly from the origin to the respective destinations. The demands are known in terms of number of cars and number of people for the respective origin-destination pair. The buses and the cars have to share the capacity of the links in the road. We aim to reach the destinations in the quickest time possible. The objective function is a little loosely defined, but we will elaborate it shortly. We provide a branch and price framework to solve the problem. We have two subproblems, one to generate the paths of the private cars and the other to generate paths of people. For the time-expanded formulation, we need to determine an upper bound on the value of T. A loose bound on this value be obtained by individually bounding the times for buses and cars separately and adding them together. The implication is we serially route them one after the other and this is still a feasible solution to the problem. This will also help us obtain an initial feasible solution to our branch and price procedure. We discuss the procedure to obtain individual time bounds later.

f_p (x_p) is a binary variable indicating whether path p is used to satisfy the demand of the corresponding origin destination pair. b_t is binary variable with value 1 if a buses tour t is used and 0 otherwise. $\alpha_1(ij,p)(\alpha_2(ij,p))$ is an indicator variable with value 1 if arc ij is in bus path(car path) p and 0 otherwise. $\beta(ij,t)$ is an indicator variable with value is 1 if arc ij is in tour t and 0 otherwise. $\gamma_1(st,p)(\gamma_2(st,p))$ is an indicator variable with value 1 if a bus path(car path) p has origin and destination as s and t respectively and 0 otherwise. Let P_1 and P_2 be the sets of all bus paths and car paths respectively and T be the set of all bus tours. B is the capacity of a bus. OD_1 and OD_2 are the sets of origins-destination pairs corresponding to buses and cars respectively. d^1_{st} and d^2_{st} is the demand of people using buses and cars respectively from origin s to destination t. Finally, b_t is a binary variable with value 1 if a tour is picked and 0 otherwise.

$$\text{Minimize} \quad \sum_{p \in P_1} c_p x_p + \sum_{p \in P_1} c_p f_p \tag{24}$$

s.t.

$$\sum_{p_1 \in P_1} \gamma_1(st,p) x_p = 1, \forall st \in OD_1 \tag{25}$$

$$\sum_{p_2 \in P_2} \gamma_2(st,p) f_p = 1, \forall s \in OD_2 \tag{26}$$

$$\sum_{p \in P_1} d^1_{st} \alpha_1(ij,p) x_p - \sum_{t \in \mathrm{T}} \beta(ij,t) B b_t \leq 0, \forall (ij) \in E \tag{27}$$

$$\sum_{p \in P_2} d^2_{st} \alpha_2(ij,p) f_p + \sum_{t \in \mathrm{T}} \beta(ij,t) b_t \leq c_{ij}, \forall (ij) \in E \tag{28}$$

$$x_p \in \{0,1\}, \forall p \in P_1 \tag{29}$$

$$f_p \in \{0,1\}, \forall p \in P_2 \tag{30}$$

$$b_t \in \{0,1\}, \forall t \in \mathrm{T} \tag{31}$$

Branch and Price Mechanism

We note that the problem has exponentially many paths in terms of the input size of the graph and we will be generating these variables in the subproblem. This is a standard approach in most of the routing and scheduling problems.

Restricted Master Problem (RMP)

The restricted master problem is obtained by relaxing constraints (28) - (30) as continuous variables and replacing the sets P_1 and P_2 by the restricted sets $P_1' \subseteq P_1$ and $P_2' \subseteq P_2$ respectively. This leads to the restricted master problem.

$$\text{Minimize} \quad \sum_{p \in P_1'} c_p x_p + \sum_{p \in P_1'} c_p f_p \tag{32}$$

s.t.

$$\sum_{p_1 \in P_1'} \gamma_1(st, p) x_p = 1, \forall st \in OD_1 \tag{33}$$

$$\sum_{p_2 \in P_2'} \gamma_2(st, p) f_p = 1, \forall st \in OD_2 \tag{34}$$

$$\sum_{p \in P_1'} d^1_{st} \alpha_1(ij, p) x_p - \sum_{t \in \mathrm{T}} \beta(ij, t) B b_t \leq 0, \forall (ij) \in E \tag{35}$$

$$\sum_{p \in P_2'} d^2_{st} \alpha_2(ij, p) f_p + \sum_{t \in \mathrm{T}} \beta(ij, t) b_t \leq c_{ij}, \forall (ij) \in E \tag{36}$$

$$0 \leq x_p \leq 0, \forall p \in P_1' \tag{37}$$

$$0 \leq f_p \leq 0, \forall p \in P_2' \tag{38}$$

$$0 \leq b_t \leq 1, \forall t \in \mathrm{T} \tag{39}$$

Let $\pi \in \mathbb{R}^{OD_1}$ be the unrestricted dual variable corresponding to constraint set (43), $\mu \in \mathbb{R}^{OD_2}$ be the unrestricted dual variable corresponding to constraint set (44), $\eta \in \mathbb{R}^E_-$ be the non-positive dual variable corresponding to the constraint set (45) and finally $\psi \in \mathbb{R}^E_-$ be the non-positive dual variable corresponding to the constraint set (46). In the pricing problem, we are interested in the reduced cost of the variable x_p and f_p. We determine the minimum reduced cost of the path flow variables in the pricing subproblems. If the minimum reduced cost corresponding to a origin destination pair is negative we add it to the restricted set (corresponding to the cars or buses) and the restricted master problem is solved again.

People-Path Subproblem

In the people-path subproblem, we determine the minimum reduced cost, $\bar{x}_p$, of a path flow variable, x_p, for people taking buses between a given origin-destination pair $st \in OD_1$. This is given by

$$\bar{x}_p = c_p - (\pi_{st} + \sum_{\forall (ij) \in E} \alpha_1(ij, p) \eta_{ij}) = -\pi_{st} + \sum_{\forall (ij) \in E} (\alpha_1(ij, p) c_{ij} - \alpha_1(ij, p) \eta_{ij}) \tag{40}$$

The cost of the path is given by the sum of the cost on arcs in the above equation. Now, the shortest path problem for all pairs of $st \in OD_1$ with the above arc costs $c_{ij} - \eta_{ij}$ is solved. If π_{st} is more than the length of a path, then it is added to the restricted path set P_1 and the RMP is solved again. We observed that the dual variable μ is negative and hence the cost on each arc is positive. Thus the shortest path problem is solvable in polynomial time.

Car-Path Subproblem

In the car-path subproblem, we are concerned with the reduced cost, $\bar{f}_p$, of car flow variables, f_p, for the pairs of origin-destination $st \in OD_2$. This given by

$$\bar{f}_p = c_p - (\mu_{st} + \sum_{\forall (ij) \in E} \alpha_2(ij,p)\psi_{ij}) = -\mu_{st} + \sum_{\forall (ij) \in E} (\alpha_2(ij,p)c_{ij} - \alpha_2(ij,p)\psi_{ij}) \tag{41}$$

We solve the shortest path, just as in people-path subproblem, but with arc cost $c_{ij} - \psi_{ij}$ and is a path costs more than μ_{st}, we add it to the restricted set P_2 and the RMP is solved again.

Branching Strategy

The branching rules is important as this determines the complexity of the pricing problem. The branching also induces some practical difficulties that needs to be explicitly handled. We address the two important problems encountered while branching.

The decision to branch occurs at a node of the branch and bound tree, when we cannot enter any more columns to the restricted sets from either subproblems and the relaxed LP solution at the current node is infeasible to the integer program. At this juncture, if any of the b_t variables are fractional, we decide to branch on them. It is easy to see that this branching will not cause any difficulty to the subproblems as the arc costs corresponding to the shortest path problems still remains positive. If none of the b_t variables are fractional and if a flow variable is fractional, a branching on the fractional path flow variable would restrict the subproblems to generate paths other than the path that was fractional. For instance, let f_p be the fractional path with value $\bar{f}_p$. We branch by adding

$$f_p \leq \lfloor \bar{f}_p \rfloor$$

to one branch and

$$f_p \geq \lceil \bar{f}_p \rceil$$

to the other branch. The difficulty now in the subproblem is that a candidate path generated for that origin-destination pair should not be the branched variable. We cannot guarantee that the shortest path problem could generate such a path. In fact after k branchings, we might have to solve the k^{th} shortest path subproblem. This is a common difficulty that arises in branch and price approaches for multicommodity flow problems. There are a few techniques in the literature that handles this issue C.Barnhart et al. (2000); Alvelos (2005); Parker and Ryan (1993); Ryan and Foster (1981). One technique is to make an arc or a set of arcs of the path that was branched as forbidden arcs in the branches. Thus the subproblem will not regenerate the path C.Barnhart et al. (2000). Another technique in the literature Alvelos (2005) to solve the problem is to branch on arc flow variable instead of path flow variable. For instance, the amount of flow of cars on an arc (ij) is given by $\sum_{p \in P_2} \alpha_2(ij,p) f_p$ and let $\bar{x}_{ij}$ be the fractional flow on the arc. So we can add the constraint

$$\sum_{p \in P_2} \alpha_2(ij,p) f_p \leq \lfloor \bar{x}_{ij} \rfloor$$

to one branch and

$$\sum_{p \in P_2} \alpha_2(ij,p) f_p \geq \lceil \bar{x}_{ij} \rceil$$

to other branch. This, however, does not guarantee positive arc costs anymore in the subproblems and they become NP-hard. This problem was overcome by adding separate variables for flows along cycles in the RMP. Thus the subproblem has to return a shortest path if available or a negative cost cycle. This could be solved in polynomial time. We employ the first method in this problem, where we make the arcs of path forbidden in the branches.

The next problem is when we arrive at a branch and bound node with the LP relaxation resulting in an infeasible solution. In an elementary branch and bound procedure we prune the search in this situation. This is, however, not possible in branch and price mechanism as we have not yet considered the entire set of columns and hence there might exist a path that has not been entered but could provide a feasible solution in the future. We take care of this issue by adding dummy paths in the initial solution with high cost that will provide us with the feasible solution.

Stabilization Techniques

The columns generation procedure, although a widely used technique to solve integer programming, has problems with convergence to an optimal solution. The problems with are usually attributed the degeneracy in the primal problem and the subproblem is solved with dual solutions with extreme values that results in generation columns that are very useful when entered in the master problem. There are a few numerical techniques available in the literature that are frequently employed to handle this situation.

Smoothing Technique

The smoothing technique involves the storing past dual solutions and generating dual solutions for the subproblems by taking the convex combinations of the current best dual solution and either past dual solutions or the current best dual solution Wentges (1997). Thus the dual solution used to solve the subproblem smoothed first and then the subproblem is solved. In this research undertaking, we store the best dual solution obtained and use it smooth the current dual solution before sending it to the subproblem.

Let $\phi_{current}$ be the current dual solution of the restricted master problem and ϕ_{best} be the , then the new dual solution is given by

$$\phi_{new} = \alpha * \phi_{best} + (1 - \alpha) * \phi_{current}, \text{ where } 0 \leq \alpha \leq 1$$

As the bound increases we increase the weight of the ϕ_{best}. It must be noted that the subproblem may not return a column for the modified dual solution. In this case, we need to decrease the weight of ϕ_{best} or make its weight equal to zero and resolve the subproblem. The method is effective and could be quite easily implemented in the current branch and price framework. We implemented this and obtained convergence in some of the instances.

Box Stabilization

The box stabilization method involves in penalizing the dual variable for leaving the box that is centered at the current best dual solution (also referred to as stability center). There are number of techniques in the literature that models this modified problem. We employ the technique proposed by du Merle et al. (1999) that involves in constraining the slack variables of the primal problem, which indirectly penalizes the dual variable from deviating from the stability center. After solving the restricted master problem we solve the subproblem. We move the stability center and the box everytime we obtain an improvement in the bound.

$$\text{Minimize} \quad \sum_{p \in P_1'} c_p x_p + \sum_{p \in P_1'} c_p f_p + \delta^+ y^+ - \delta^- y^- \tag{42}$$

s.t.

$$\sum_{p_1 \in P_1'} \gamma_1(st, p) x_p + y_1^+ - y_1^- = 1, \forall st \in OD_1 \tag{43}$$

$$\sum_{p_2 \in P_2'} \gamma_2(st, p) f_p + y_2^+ - y_2^- = 1, \forall st \in OD_2 \tag{44}$$

$$\sum_{p \in P_1'} d_{st}^1 \alpha_1(ij, p) x_p - \sum_{t \in \mathrm{T}} \beta(ij, t) B b_t + y_3^+ - y_3^- \leq 0, \forall (ij) \in E \tag{45}$$

$$\sum_{p \in P_2'} d_{st}^2 \alpha_2(ij, p) f_p + \sum_{t \in \mathrm{T}} \beta(ij, t) b_t + y_4^+ - y_4^- \leq c_{ij}, \forall (ij) \in E \tag{46}$$

$$y^+ \leq \epsilon^+ \tag{47}$$

$$y^- \leq \epsilon^- \tag{48}$$

$$0 \leq x_p \leq 0, \forall p \in P_1' \tag{49}$$

$$0 \leq f_p \leq 0, \forall p \in P_2' \tag{50}$$

$$0 \leq b_t \leq 1, \forall t \in \mathrm{T} \tag{51}$$

In the above formulation, we add the slack and surplus variables to the primal constraints and we penalize the variables in the objective function. We also restrict the slack and surplus variables, which provides the penalty for dual variables from deviating from the stability center. At the end of every iteration we update the stability center if the dual bound improves and change the right hand side and cost coefficients of the slack and surplus variables accordingly. We are currently implementing this strategy and this could be realized within our current framework.

Bundle methods

The bundle method treats the column generation problem as cutting plane method in the dual problem and adds cutting planes to the restricted Lagrangian dual problem and solves the restricted dual every iteration. We provide the information for the sake of completeness of the study. The current implementation framework cannot accommodate this technique.

Interior Point Stabilization

Interior point stabilization is a recent stabilization technique, which has shown great promise. We are currently developing the theory to accommodate our model to incorporate a primal-dual interior-point stabilization. The idea is to generate dual interior point solution that prevents the subproblem to work with dual extreme point solutions and hence provide faster convergence.

Preliminary Results

We tested grid graphs of size 25 to 400 nodes. Table 3 enumerates the instances we tested. The largest instance tested was grid graph with 400 nodes and 1,520 edges with 100 cars and 40 buses.

Additionally, in order to test the robustness of the code we tested a few online benchmark instances for multicommodity flow problems Larsson and Yuan (2004) for four planar graphs and results are provided in table 4. We generated one dummy bus tour and one bus commodity for each of the instances.

Table 3: Branch & Price model tested on grid graphs

#	Nodes	Arcs	Cars	Buses	Tours	CPU Time
1	25	80	6	6	3	0.013
2	25	80	10	10	4	0.064
3	25	80	20	10	4	0.174
4	25	80	15	15	4	0.089
5	100	360	5	5	4	0.138
6	100	360	10	10	5	1.161
7	100	360	20	10	5	1.063
8	100	360	20	20	7	0.8
9	225	840	10	10	7	5.637
10	225	840	20	20	10	1.205
11	400	1520	100	40	14	52.503

Table 4: Branch & Price model tested on planar graphs with one bus commodity

#	Nodes	Arcs	Cars	CPU Time
1	30	150	92	0.08
2	50	250	267	0.177
3	80	440	543	0.509
4	100	532	1085	1.605

Conclusions

The models currently available in the literature are usually customized for the evacuation of specific regions in geography tailored to its needs. The models have their relative advantages and disadvantages, but precise to their needs. It is rather tedious to generate a unified model that could be used in all situations. Inclusion of several features impacts the complexity of the model and hence the computational speed. On the other hand, simplifying a model would compromise the precision of the model. However, the models have some common overlapping features that we highlighted in this report. We saw that the hybrid models could be reasonably accurate and precise by exploiting the relative advantages of simulation and optimization techniques. We provided broad classification of evacuation models based on simulation or optimization methods. Furthermore, we identified the factors considered by these optimization models and commented on the approaches. We highlighted the features that will have a significant effect on the travel time, namely intermodal transportation, dynamic traffic demand estimation and contra flows or lane reversals in case of a wide-area evacuation. Models incorporating intermodal transportation and contra flows demonstrated the improvement in evacuation efficiency compared to the traditional models. Also, static demand model have become obsolete and dynamic demand is necessary for practical realization of the models. Some areas that needs attention are optimization problems to establish alternate evacuation paths for incident managements. Critical node detection and traffic management on critical links are studies that might improve the efficiency of the evacuation and also might give an indication of the necessary links that we could focus on contraflows. Also heuristic exploration of optimization techniques could significantly reduce the computational speed. Research in the field of clustering of nodes and zonal division of network is very limited. This might shed light in performing evacuation over a smaller aggregated network helping in computational efficiency.

We formally introduced the contraflow problem that has applications in emergency transportation management. Several classic network flow problems are studied, including static and dynamic networks. A polynomial time algorithm for the dynamic contraflow problem with single source and single sink is given, together with an NP-completeness proof for the dynamic transhipment contraflow problem. The hardness of the contraflow problem with arc reversal cost was also indicated.

We provided a branch and price framework to solve a bimodal multicommodity flow problem. We consider cars and buses as two modes of transportation and we obtained optimal paths for cars and identified the bus routes needed for transportation. We tested the model on grid graphs of sizes up to 400 nodes. As a future work, we need to employ heuristic methods for the subproblems, develop procedures that would provide good lower bound for the branch and bound. We are currently in the process of implementing a box stabilization technique in order to accelerate the convergence of the branch and price procedure that would enable us to test much larger instances and we need to compare the computational performance against other stabilization techniques.

Although the theory and algorithms developed are exact methods (the proof of correctness itself is a validation), a good validation approach from a practical sense is necessary for testing for negating the assumptions made in the model and check the computational efficiency on large scale realtime instances. More importantly, the applicability of the the algorithm and hence the results to real-life problems would also be satisfactory at this stage. This would require some additional effort with which it could be accomplished. The models were developed addressing specific events arising in emergency situations such as evacuating stadiums, cities and buildings, but the model could be extended to accommodate more general instances and provide large-scale transportation solutions. These could be handled with minor changes to input and algorithmic parameters depending on the situation.

Related Publications

1. A. Arulselvan, L. Elefteriadou, and P.M. Pardalos (2008). A Review in Evacuation Problems. Center for Multimodal Transportation for Congestion Mitigation and Center for Applied Optimization, University of Florida.
2. S. Rebennack, A. Arulselvan, L. Elefteriadou, P.M. Pardalos (2008), Complexity Issues of Maximum Flow problems with Arc Reversal. Journal of Combinatorial Optimization (accepted).

Under Preparation
3. A. Arulselvan, L. Elefteriadou, P.M. Pardalos. Multimodal solutions for evacuation problem.

References

Abbas-Turki, A., Bouyekhf, R., Grunder, O., Moudni, A. E., 2004. On the line planning problems of the hub public-transportation networks. International Journal of Systems Science 35 (12), 693–706.

Aerde, M. V., Rakha, H., Paramahamsan, H., 2003. Estimation of o-d matrices: The relationship between practical and theoretical considerations. In: In the proceedings of 82nd TRB Annual Meeting. pp. 122–130.

Ahuja, R. K., Magnati, T. L., Orlin, J. B., 1993. Network Flows: Theory, Algorithms, and Applications. Prentice Hall, Englewood Cliffs, New Jersey.

Algers, S., Bernauer, E., Boero, M., Breheret, L., Taranto, C. D., Dougherty, M., Fox, K., 1997. Review of micro-simulation models. Tech. rep., SMARTEST Project Deliverable D3, Institute for Transport Studies, University of Leeds.

Alvelos, F., 2005. Branch-and-Price and Multicommodity Flows. Ph.D. thesis, Universidade do Minho, http://repositorium.sdum.uminho.pt/dspace/bitstream/1822/2736/1/falvelos_phd.pdf.

Antoniou, C., Ben-Akiva, M., Bierlaire, M., Mishalani, R., 1997. Demand simulation for dynamic traffic assignment. In: Presented at the 8th IFAC Symposium on Transportation Systems. pp. 114–118.

Arulselvan, A., Elefteriadou, L., Pardalos, P., 2004. A survey on evacuation problems. Tech. rep., Center for Multimodal Transportation and Congestion MItigation, University of Florida.

Ashok, K., Ben-Akiva, M., 1993. Dynamic o-d matrix estimation and prediction for real-time traffic management systems. In: Daganzo, C. (Ed.), International Symposium on Transportation and Traffic Theory. Vol. 2337 of Transporation and Traffic Theory. Elsevier Science Publishing Company Inc., pp. 36–53.

Bakuli, D., Smith, J., 1996. Resource allocation in state-dependent emergency evacuation networks. European Journal of Operational Research 89 (3), 543–555.

Ban, J., Liu, H., Ran, B., September 2005. A link based quasi-variational inequality model for dynamic user equilibria, towards real time traffic operations. In: Proceedings of the 8th International IEEE Conference on Intelligent Transportation Systems. pp. 458– 463.

Bern, M. W., Lawler, E. L., Wong, A. L., 1987. Linear-time computation of optimal subgraph of decomposable graphs. Journal on Algorithms 8, 216–235.

Bliemer, M., Bovy, P., 2003. Quasi-variational inequality formulation of the multiclass dynamic traffic assignment problem. Transportation Research B 37, 501–519.

Blum, J., Eskandarian, A., Oct 2004. The impact of multi-modal transportation on the evacuation efficiency of building complexes. In: Proceedings of the 7th International IEEE Conference on Intelligent Transportation Systems. pp. 702–707.

Borndörfer, R., Grötschel, M., Pfetsch, M., 2007. A column-generation approach to line planning in public transport. Transportation Science 41 (1), 123–132.

Bretherton, W., Elhaj., M., 1996. Is a reversible lane system safe. In: Compendium of technical papers for 66th ITE Annual Meeting. pp. 277–281.

Brotcorne, L., Wolf, D. D., Gendreau, M., Labb, M., 2002. A dynamic user equilibrium model for traffic assignment in urban areas. In: Gendreau, M., Marcotte, P. (Eds.), Transportation and Network Analysis: Current Trends. Kluwer Academic Publishers, pp. 49–70.
URL citeseer.ist.psu.edu/576208.html

Bryan, J., Mar 2000. Human behaviour in fire: the development and maturity of a scholarly study area. Fire and Materials 23 (6), 249–253.

Burkard, R., Dlaska, K., Klinz, B., February 1993a. The quickest flow problem. Mathematical Methods of Operations Research 37 (1), 31–58.

Burkard, R., Dlaska, K., Klinz, B., 1993b. The quickest flow problem. Mathematical Methods of Operations Research 37 (1), 31–58.

Carey, M., 1986a. A constraint qualification for a dynamic traffic assignment. Transportation Science 20 (1), 55–58.

Carey, M., 1986b. Optimal time-varying flows on congested networks. Operations Research 14 (1), 55–58.

C.Barnhart, Hane, C., Vance, P., 2000. Using branch-and-price-and-cut to solve origin-destination integer multicommodity flow problems. Oper. Res. 48 (2), 318–326.

Chalmet, L., Francis, R., Saunders, P., 1982. Network models for building evacuation. Management Science 28, 86–105.

Chang, E., Ziliaskopoulos, A., 2004. Formulation of analytical time varying intermodal person trip assignment model. Transportation Research Record 1882, 1–9.

Chen, A., Kongsomsaksakul, S., Zhou, Z., 2007. Assessing network vulnerability using combined travel demand model. In: 86th Transportation Research Board Annual Meeting, Washington, D.C. pp. 07–2667.

Chen, X., September 2006. Agent based simulation of evacuation strategies under different road network structures. Tech. rep., University Consortium of Geographic Information Science.

Chen, X., Zhan., B., 2006. Agent-based modeling and simulation of urban evacuation relative effectiveness of simultaneous and staged evacuation strategies. Journal of Operations Research Society 59 (1), 25–33.

Cheriyan, J., Maheshwari, S. N., 1989. Analysis of preflow push algorithm for maximum network flow. SIAM Journal on Computing 18, 1057–1086.

Chiu, Y., Villalobos, J., Gautam, B., Zheng, H., January 2007. Modeling no-notice mass evacuation using a dynamic traffic flow optimization model. IIE Transactions 39 (1), 83–94.

Church, R., Sexton, R., April 2002. Modeling small area evacuation: Can existing transportation infrastructure impede public safety? Tech. rep., Caltrans Testbed Center for Interoperability Task Order 3021.

Cova, T., Johnson, J., 2002. Microsimulation of neighborhood evacuation in urban wildland interface. International Journal of Geographical information System 11 (8), 763–784.

Dafermos, S., 1980. Traffic equilibria and variational inequalities. Transportation Science 14, 42–54.

Daganzo, F., 1994. The cell transmission model: A dynamic representation of highway traffic consistent with the hydrodynamic theory. Transportation Research Part B 28B (4), 269–287.

de Silva, F., Eglese, R., Pidd, M., 2003. Evacuation planning and spatial decision making: designing effective spatial decision support systems through integration of technologies. IGI Publishing, Hershey, PA, USA, pp. 358–373.

du Merle, O., Villeneuve, D., Desrosiers, J., Hansen, P., 1999. Stabilized column generation. Discrete Mathematics 194, 229237.

Fleischer, L., Skutella, M., 2002. The quickest multicommodity flow problem. Lecture Notes in Computer Science. Springer, pp. 36–53.

Ford, F. R., Fulkerson, D. R., 1962. Flows in Networks. Princeton University Press, Princeton, New Jersey.

Ford, L., Fulkerson, D., 1958. Constructing maximal dynamic flows from static slows. Operations Research 6, 419–433.

Friesz, T., Bernstein, D., Smith, T., Tobin, R., Wie, B., 1993. A variational inequality formulation of the dynamic network user equilibrium problem. Operations Research 41 (1), 179–191.

Fu, H., Wilmot, C., 2004. A sequential logit dynamic travel demand model for hurricane evacuation. Transportation Research Record 1882, 19–26.

Fu, H., Wilmot, C., 2007. Modeling the hurricane evacuation response curve. TRANSPORTATION RESEARCH RECORD 2022, 94–102.

Garey, M. R., Johnson, D. S., 1979. Computers and Intractability - A Guide to the Theory of NP-Completeness. W. H. Freeman and Company, New York.

Gary, M., Johnson, D., 1979. A guide to Theory of NP-Completeness. W.H. Freeman and Company.

Golani, H., Waller, S., 2004. Combinatorial approach multiple destination user optimal dynamic traffic assignment. Transportation Research Record 1882, 70–78.

Goldberg, A. V., Tarjan, R. E., 1989. Finding minimum-cost circulations by cancelling negative cycles. Journal of ACM 36, 873–886.

Gomes, G., Munoz, L., Yi, C., Toy, Cinnamon, S., Horowitz, R., Alvarez, L., 2001. Meso-microscale traffic simulation of an ahs control architecture. In: Proceedings of the American Control Conference. pp. 1806–1811.

Goossens, J., Hoesel, C., Kroon, L., 2002. On solving multi-type line planning problems. METEOR Research Memorandum RM/02/009, University of Maastricht, Maastricht, The Netherlands.
URL citeseer.ist.psu.edu/goossens02solving.html

Gross, J. L., Yellen, J., 2003. Handbook of Graph Theory (Discrete Mathematics and Its Applications). CRC, New York.

Guisewite, G. M., Pardalos, P. M., 1990. Minimum Concave-Cost Network Flow Problems: Applications, Complexity, and Algorithms. Annals of Operations Research 25, 75–100.

Gwynne, S., Galea, E. R., Owen, M., Lawrence, P., Filippidis, L., 1999. A review of the methodologies used in the computer simulation of evacuation from the built environment. Building and Environment 34 (6), 741–749.

Hajek, B., Ogier, R. G., 1984. Optimal dynamic routing in communication networks with continuous traffic. Networks 14, 457–487.

Hamza-Lup, G. L., Hua, K., Lee, M., Peng, R., 2004. Enhancing intelligent transportation systems to improve and support homeland security. In: Proceedings of the 7th International IEEE Conference on Intelligent Transportation Systems. pp. 250–255.

Hartman, P., Stampacchia, G., 1966. On some nonlinear elliptic diferential functional equations. Acta Mathematica 115, 271–310.

Helbing, D., Farkas, I., Vicsek, T., 2000. Simulating dynamical features of escape panic. Nature 407, 487.

Ho, J., 1980. A successive linear optimization approach to dynamic traffic assignment. Transportation Science 14 (4), 295–305.

Holguin-Veras, E., Perez, N., Ukkusuri, S., Wachtendorf, T., Brown, B., 2007. Emergency logistics issues affecting response to hurricane katrina: Synthesis and preliminary suggestions for improvement. Transportation research record 2022, 76–82.

Hoppe, B., Tardos, E., 1994. Polynomial time algorithms for some evacuation problems. In: SODA '94: Proceedings of the fifth annual ACM-SIAM symposium on Discrete algorithms. Society for Industrial and Applied Mathematics, pp. 433–441.

Hoppe, B., Tardos, E., 1995. The quickest transshipment problem. In: SODA '95: Proceedings of the sixth annual ACM-SIAM symposium on Discrete algorithms. Society for Industrial and Applied Mathematics, pp. 512–521.

Hoppe, B. E., 1995. Efficient Dynamic Network Flow Algorithms. Ph.D. thesis, Cornell University.

Kim, D., Pardalos, P. M., 2000. Dynamic Slope Scaling and Trust Interval Techniques for Solving Concave Piecewise Linear Network Flow Problems. Networks 35 (3), 216–222.

Kim, S., Shekhar, S., 2005. Contraflow Network Reconfiguration for Evaluation Planning: A Summary of Results. In: Proceedings of the 13th annual ACM international workshop on Geographic information systems. pp. 250–259.

Klinz, B., Woeginger, G., 2004. Minimum-cost dynamic flows: The series-parallel case. Networks 43, 153–162.

Krumke, S., Noltemeier, H., Schwarz, S., Wirth, H., Ravi, R., 1998. Flow Improvement and Network Flows with Fixed Costs. In: In Proceedings of the International Conference of Operations Research Ziirich (0R'98). pp. 158–167.

Kuligowski, E., Peacock, R., July 2005. Review of building evacuation models. Tech. rep., National Institute of Standards and Technology.

Larsson, T., Yuan, D., 2004. An augmented lagrangean algorithm for large scale multicommodity routing. Computational Optimization and Applications 27 (2), 187–215.

Liu, H., He, X., Ban, X., 2007. Cell-based many-to-one dynamic system optimal model and its heuristic solution method for emergency evacuation. In: 86th Transportation Research Board Annual Meeting, Washington, D.C. pp. Paper 07–2261, 20 pages.

Liu, Y., Lai, X., , Chang, G.-L., 2006a. A cell-based network optimization model for staged evacuation planning under emergencies. Transportation Research Record 1964, 127–135.

Liu, Y., Lai, X., Chang, G.-L., october 2006b. Two-level integrated optimization system for planning of emergency evacuation. Journal of transportation Engineering 10 (10), 800–807.

Liu, Y., Zou, N., Chang, G.-L., 2005. An integrated emergency evacuation system for real-time operations - a case study of ocean city, maryland under hurricane attacks. In: In the proceedings of Intelligent Transportation Systems, IEEE. pp. 464– 469.

Megiddo, N., 1979. Combinatorial optimization with rational objective functions. Mathematics of Operations Research 4, 414–424.

Melkonian, V., 2007. Flows in dynamic networks with aggregate arc capacities. Information Processing Letters 101 (1), 30–35.

Merchant, D., Nemhauser, G., August 1978. A model and an algorithm for the dynamic traffic assignment problems. Transportation science 12 (3), 183–199.

Miller-Hooks, E., Patterson, S., 2004. On solving quickest time problems in time-dependent, dynamic networks. Journal of Mathematical Modelling and Algorithms 3 (1), 39–71.

Minieka, E., 1973. Maximal, lexicographic, and dynamic network flows. operations research. Transportation Research Record 21, 517–527.

Mitchell, S., Radwan, A., 2006. Heuristic priority ranking of emergency evacuation staging to reduce clearance time. Transportation Research Record 1964, 219–228.

Moriarty, K., Ni, D., Collura, J., 2007. Modeling traffic flow under emergency evacuation situations: Current practice and future directions. In: 86th Transportation Research Board Annual Meeting, Washington, D.C. pp. Paper 07–0745, 15 pages.

Murray, P., Mahmassani, H., 2004. Model for household trip chain sequencing in an emergency evacuation. In: In the proceedings of 82nd TRB Annual Meeting. pp. 21–29.

Nagurney, A., 1999. Network Economics: A Variational Inequality Approach. Springer.

Opasanon, S., 2004. On finding paths and flows in multicriteria, stochastic, and time-varying networks. Ph.D. thesis, Department of Civil Engineering and Environmental Engineering, University of Maryland, College Park.

Orlin, J. B., 1983. Maximum-throughput dynamic network flows. Mathematical Programming 27, 214–231.

Ozbay, K., Ozguven, E., 2007. A stochastic humanitarian inventory control model for disaster planning. In: 86th Transportation Research Board Annual Meeting, Washington, D.C. pp. 63–75.

Parker, M., Ryan, A., 1993. A column generation algorithm for bandwidth packing. Telecommunications Systems 2, 185–195.

Payne, H., 1971. Models of freeway traffic and control. In: Math. Models Publ. Sys., Simul. Council Proc. Vol. 28. pp. 51–61.

Peeta, S., Ziliaskopoulos, A., 2001. Foundations of dynamic traffic assignment: Past, present and the future. Networks and Spatial Economics 1 (3-4), 233–265.

Pfetsch, M., R.Borndrfer, 2005. Routing in line planning for public transport. In: Hans-Dietrich Haasis, H. K., Schnberger, J. (Eds.), Operations Research Proceedings. pp. 405–410.

Pidd, M., de Silva, F., Eglese, R., Dec 1993. Cemps: A configurable evacuation management and planning system. In: Proceedings of the Winter Simulation Conference. pp. 1319–1323.

Raciti, F., L.Scrimali, 2004. Time-dependent variational inequalities and applications to equilibrium problems. J. of Global Optimization 28 (3-4), 387–400.

Radwan, A., Hoebeika, A., Sivasailam, D., September 1985. A computer simulation model for rural network evacuation under natural disaster. Institute of Transportation Engineers Journal 55 (9), 25–30.

Radwan, E., Mitchell, S., Yildirim, G., April 2005. Framework for modeling emergency evacuation. Tech. rep., Center for Advanced Transportation Systems Simulation. University of Central Florida.

Ran, B., Boyce, D., 1996. Modelling Dynamic Transportation networks, 2nd Edition. Springer.

Rebennack, S., Arulselvan, A., Elefteriadou, L., Pardalos, P., 2008. Complexity analysis for maximum flow problems with arc reversals, submitted.

Ryan, D., Foster, B. A., 1981. An integer programming approach to scheduling. In: Wren, A. (Ed.), Computer Scheduling of Public Transport Urban Passenger Vehicle and Crew Scheduling.

Sakakibara, H., Kajitani, Y., Okada, N., 2004. Road network robustness for avoiding functional isolation in disasters. Journal of transportation Engineering 130 (5), 560–567.

Santos, G., Aguirre, B., 2004. A critical review of emergency evacuation simulation models. In: Proceedings of Building Occupant Movement during Fire. National Institute of Standards and Technology, pp. 27–52.

Sattayhatewa, P., Ran, B., 2004. Developing a dynamic traffic management model for nuclear power plant evacuation. In: 83rd Transportation Research Board Annual Meeting, Washington, D.C.

Sbayti, H., Mahmassani, H., 2006. Optimal scheduling of evacuation operations. Transportation Research Record 1964, 238–246.

Sheffi, Y., Mahmassani, H., Powell, W., 1980. NETVAC: A transportation Network Evacuation Model. Center for Transportaion Studies, MIT.

Sisiopiku, V., Jones, S., Sullivan, A., Patharkar, S., Tang, X., April 2004. Regional traffic simulation for emergency preparedness. Tech. rep., University Transportation Center for Alabama, The University of Alabama, The University of Alabama in Birmingham, and The University of Alabama in Huntsville.

Smith, J., Jan 1987. Qnet-c: An interactive graphics computer program for evacuation planning. In: Proceedings of the conference on Emergency plannning, SCS multiconference, 14-16. pp. 19–24.

Smith, J., 1993. Multiobjective routing in stochastic evacuation network. In: D-Z. Du, P. P. (Ed.), Network Optmization Problem. Vol. 2. World Scientific, pp. 263–281.

Smith, J., Towsley, D., 1981. The use of queueing networks in the evacuation of egress from buildings. Environment and Planning B-8, 125–139.

Stathopoulos, A., Tsekeris, T., 2004. Enhanced dynamic origin-destination matrix updating with long term flow information. Transportation Research Record 1882, 159–166.

Tagliaferri, A. P., 2005. Use and comparison of traffic simulation models in the analysis of emergency evacuation conditions. Ph.D. thesis, Department of Civil Engineering, North Carolina State University.

Talebi, K., Smith, J., 1985. Stochastic network evacuation models. Computers and Operations Research 12 (6), 559–577.

Theodoulou, G., Wolshon, B., 2004. Alternative methods to increase the effectiveness of freeway contraflow evacuation. The Journal of Transportation Research Board 1865, 48–56.

Tuite, P., Mahmassani, H., 2004. Methodology for determining vulnerable links in transportation network. Transportation Research Record 1882, 88–96.

Tuydes, H., Ziliaskopoulos, A., 2006a. Network re-design to optimize evacuation contraflow. Transportation Research Record: Journal of the Transportation Research Board 1964, 157–168.

Tuydes, H., Ziliaskopoulos, A., 2006b. Tabu-based heuristic approach for optimization of network evacuation contraflow. Transportation Research Record 1964, 157–168.

Waller, S., Ziliaskopoulos, A., 2001. Stochastic dynamic network design problem. Transportation Research Records 1771, 106–113.

Wentges, P., 1997. Weighted dantzig-wolfe decomposition for linear mixed-integer programming. Int. Trans. Oper. Res. 4 (2), 151162.

Wilkinson, W. L., 1971. An algorithm for universal maximal dynamic flows in a network. OPerations Research 19, 1602–1612.

Williams, B., Tagliaferri, A., Meinhold, S., Hummer, J., Rouphail, N., 2007. Simulation and analysis of freeway lane reversal for coastal hurricane evacuation. J. Urban Plng. and Devel. 133 (1), 61–72.

Wolshon, B., Urbina, E., Levitan, M., 2001. National review of hurricane evacuation plans and policies. louisiana. Tech. rep., State University Hurricane Center.

Yuan, F., Han, L., 2005. Evacuation modeling and operations using dynamic traffic assignment and most-desirable destination approaches. In: Proceedings of TRB 84th Annual Meeting, Transportation Research Board. p. CDROM.

Yuan, F., Han, L., Chin, S.-M., Hwang, H., 2006. Proposed framework for simultaneous optimization of evacuation traffic destination and route assignment. Transportation Research Record 1964, 50–58.

Yuan, F., Han, L., Chin, S.-M., Hwang, H., 2007. Does noncompliance with route and destination assignment compromise evacuation efficiency? In: 86th Transportation Research Board Annual Meeting, Washington, D.C. pp. paper 07–2396, 25 pages.

Zhou, X., Erdogan, S., Mahmassani, H., 2006. Dynamic origin-destination trip demand estimation for subarea analysis. In: 85th Transportation Research Board Annual Meeting, Washington, D.C. pp. 176–184.

Ziliaskopoulos, A., February 2000. A linear programming model for the single destination system optimum dynamic traffic assignment problem. Transportation science 34 (1), 37–49.

Zou, N., Yeh, S.-T., Chang, G.-L., Marquess, A., Zezeski, M., 2005. Simulation-based emergency evacuation system for ocean city, maryland, during hurricanes. Transportation Research Record 1922, 138–148.